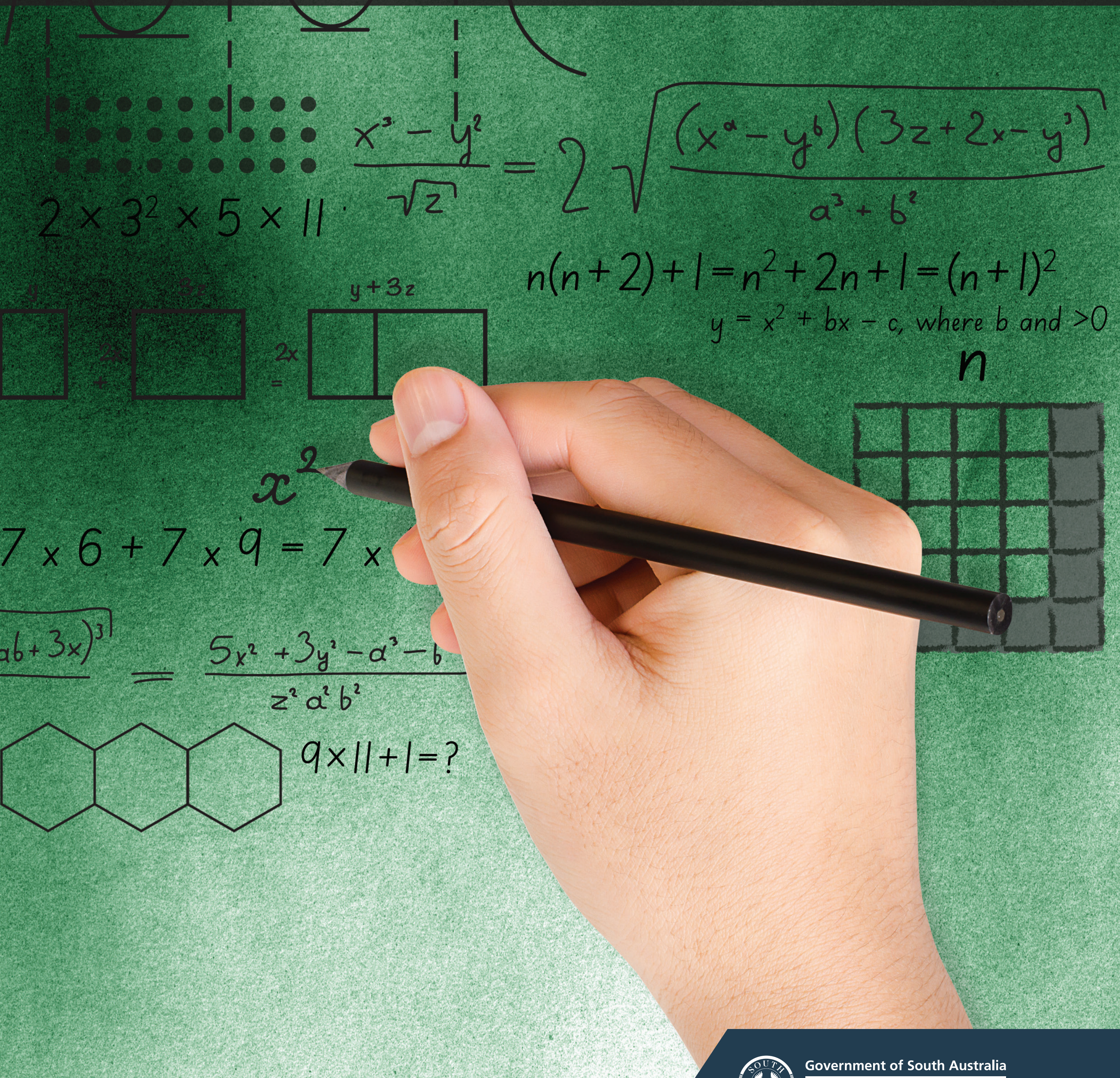
Patterns and algebra: Year 9

MATHEMATICS CONCEPTUAL NARRATIVE

Leading Learning: Making the Australian Curriculum work for us
by bringing CONTENT and PROFICIENCIES together



www.acleadersresource.sa.edu.au

Government of South Australia
Department for Education

Contents

Resource key

The '**AC**' icon indicates the Australian Curriculum: Mathematics content description(s) addressed in that example.

The '**From tell to ask**' icon indicates a statement that explains the transformation that is intended by using the task in that example.

More information about 'Transforming Tasks':
http://www.acleadersresource.sa.edu.au/index.php?page=into_the_classroom

Look out for the purple pedagogy boxes, that link back to the SA TfEL Framework.

The '**Bringing it to Life (BitL)**' tool icon indicates the use of questions from the *Leading Learning: Making the Australian Curriculum Work for Us* resource.

Bringing it to Life (BitL) key questions are in bold orange text.

Sub-questions from the BitL tool are in green medium italics – these questions are for teachers to use directly with students.

More information about the 'Bringing it to Life' tool:
http://www.acleadersresource.sa.edu.au/index.php?page=bringing_it_to_life

Throughout this narrative—and summarised in **'Patterns and algebra' from Foundation to Year 10A** (see page 19)—we have colour coded the AC: Mathematics year level content descriptions to highlight the following curriculum aspects of working with patterns and algebra:

- Copy, continue and create patterns
- Investigate and describe number patterns
- Use variables to represent numbers and create algebraic expressions
- Simplify and identify equivalent algebraic expressions by extending and applying laws and properties of numbers
- Use algebraic thinking and processes to solve problems.

What the Australian Curriculum says about 'Patterns and algebra'

Content descriptions

Strand | Number and algebra.

Sub-strand | Patterns and algebra.

Year 9 ◆ | ACMNA212
Students extend and apply the index laws to variables, using positive integer indices and the zero index.

Year 9 ◆ | ACMNA213
Students apply the distributive law to the expansion of algebraic expressions, including binomials, and collect like terms where appropriate.

Year level descriptions

Year 9 ◆ | Students simplifying a range of algebraic expressions.

Year 9 ◆ | Students applying the index laws to expressions with integer indices.

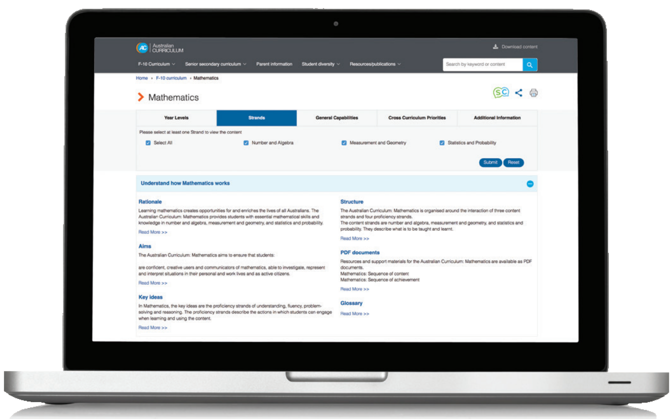
Achievement standards

Year 9 ◆ | Students expand binomial expressions.

Numeracy continuum

Recognising and using patterns and relationships

End of Year 10 ◆ | Students explain how the practical application of patterns can be used to identify trends (Recognise and use patterns and relationships).



Source: ACARA, Australian Curriculum: Mathematics

Working with Patterns and algebra

Important things to notice about this sub-strand of the Australian Curriculum: Mathematics and numeracy continuum

What we are building on and leading towards in Year 9 ‘Patterns and algebra’

In Year 7 the concept of a variable as a way of representing numbers using letters is introduced. Students still create and evaluate expressions, just as they had in previous years; but now they will create algebraic expressions instead of number sentences. This transition from writing number sentences to algebraic expressions requires the students to apply the laws of arithmetic to algebra. This application begins in Year 7 when students will apply the associative [a+(b+c) = (a+b)+c] and commutative laws [a+b = b+a] to algebraic expressions in order to simplify them.

In Year 8 students begin to expand algebraic expressions applying the distributive law [a x (b+c) = a x b + a x c] to algebra. They simplify more complex expressions that may include brackets and any or all of the four operations. The relationship between expansion and factorisation is also explored as students begin to factorise using a numerical factor.

In Year 9 the application of the laws of arithmetic to algebra continues and now includes the index laws; but only using positive or zero integers at this stage. Students continue to apply the distributive law, but to more complex expressions, including binomials.

In Year 10 students continue to simplify, factorise and expand algebraic expressions. Factorisation at this level will now include taking out an algebraic factor of monic algebraic expressions and indices could include positive or negative integers. Simplification now includes algebraic fractions with all four operations.

In Year 10A students apply long division to polynomials and they investigate the relationship between this, and the factor and remainder theorems.

- **Notice that** not all Year 9 learners will have multiplicative thinking and proportional reasoning, and this limits their conceptual understanding of algebraic principles. When designing learning at this level, provide concrete, practical and numerical examples to connect the students existing knowledge to the more abstract concepts. Encourage students to visualise the concept and verbalise their thinking before you present them with, or expect them to provide, abstract algebraic representations.
- There is not much focus on using algebraic thinking and processes to solve problems in ‘Patterns and algebra’ content descriptions, but it is implicit in problem-solving in all other sub-strands at this level.

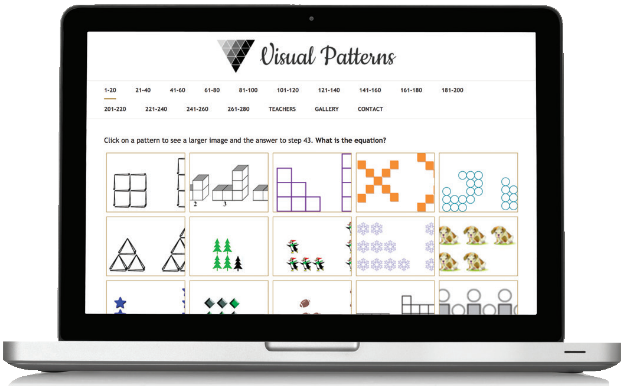
Engaging learners

Classroom techniques for teaching Patterns and algebra

Visual patterns

This website has multiple visual patterns that students can consider and describe in a way that makes sense to them. Students will have multiple interpretations of each image, promoting multiple ways of visualising, solving problems and stimulating dialogue. There is no right, or wrong answer and all learners can participate while you stay in the 'opinion space'. They can be used as lesson starters.

Visual patterns can be found at:
http://www.visualpatterns.org/



Source: *Visual Patterns*, visualpatterns.org, 2017

Counter-intuitive experiences

Counter-intuitive experiences intrigue students who want to make sense of what they have seen. This is a way to cognitively engage students to explore the phenomena more closely and use or be convinced, by the mathematics that explains it.

The Flash Mind Reader is an impressive 'confidence trick'.

An electronic version of the game can be found at:
http://www.flashlightcreative.net/swf/mindreader/

Source: *The Original Flash Mind Reader*, flashlight creative

From tell to ask

Transforming tasks by modelling the construction of knowledge (Examples 1–5)

The idea that education must be about more than transmission of information that is appropriately recalled and applied, is no longer a matter for discussion. We know that in order to engage our students and to support them to develop the skills required for success in their life and work, we can no longer rely on a 'stand and deliver' model of education. It has long been accepted that education through transmission of information has not worked for many of our students. Having said this, our classrooms do not necessarily need to change beyond recognition. One simple, but highly effective strategy for innovation in our classrooms involves asking ourselves the question:

What information do I need to tell my students and what could I challenge and support them to develop an understanding of for themselves?

For example, no amount of reasoning will lead my students to **create the names for factorisation (such as the difference of two squares)** for themselves. They need to receive this information in some way. However, it is possible my students can be challenged with questions that will result in them **identifying the relationships between numbers and their factors**, so I don't need to instruct that information.

At this stage of development, students can **develop an understanding of factorising and expanding algebraic expressions, while using and practising their knowledge of arrays and number facts**. When teachers provide opportunities for students to **identify and represent the factors of numbers**, they require their students to generalise from the patterns they have observed. Telling students algebraic rules removes this natural opportunity for students to make conjectures and verify and apply connections that they notice. Using questions such as the ones described here, supports teachers to replace 'telling' the students information, with getting students to notice for themselves.

When we are feeling 'time poor' it's tempting to believe that it will be quicker to tell our students a formula, rather than ask a question (or series of questions) and support them to establish a formula for themselves. Whether this is true or not really depends on what we have established as our goal. If our goal is to have students recall and apply a particular rule or theorem during the current unit of work, then it probably is quicker to tell them the rule and demonstrate how to apply it. However, when our goal extends to wanting students to develop conceptual understanding, to learn to think mathematically, to have a self-concept as a confident and competent *creator and user* of mathematics, then telling students the rule is a false economy of time.

When we challenge our students to **establish a theorem**, we model that algebra can be powerful and useful. We provide our students with an authentic context for working algebraically. Telling students formulae removes this opportunity for students to generalise.

Teachers can support students to understand the factorisation of numbers by asking questions as described in the **Understanding** proficiency: **What patterns/connections/relationships can you see?** The intent of this question is to promote learning design that intentionally plans for students to develop a disposition towards looking for patterns, connections and relationships.

Curriculum and pedagogy links

The following icons are used in each example:

The '**AC**' icon indicates the Australian Curriculum: Mathematics content description(s) addressed in that example.

The '**Bringing it to Life (BitL)**' tool icon indicates the use of questions from the *Leading Learning: Making the Australian Curriculum Work for Us* resource.

The Bringing it to Life tool is a questioning tool that supports teachers to enact the AC: Mathematics Proficiencies: http://www.acleadersresource.sa.edu.au/index.php?page=bringing_it_to_life

The '**From tell to ask**' icon indicates a statement that explains the transformation that is intended by using the task in that example.

This idea of moving 'From tell to ask' is further elaborated (for Mathematics and other Australian Curriculum learning areas) in the 'Transforming Tasks' module on the *Leading Learning: Making the Australian Curriculum work for us* resource: http://www.acleadersresource.sa.edu.au/index.php?page=into_the_classroom

Look out for the purple pedagogy boxes, that link back to the SA TfEL Framework.

From tell to ask examples	
Example 1: Reconstruct a rectangle – taking out the common factor Students apply the distributive law to the expansion of algebraic expressions, including binomials, and collect like terms where appropriate.	ACMNA213 ◆
Example 2: Prime factorisation products – index laws Students extend and apply the index laws to variables, using positive integer indices and the zero index.	ACMNA212 ◆
Example 3: One above/one below – difference of two squares Students apply the distributive law to the expansion of algebraic expressions, including binomials, and collect like terms where appropriate.	ACMNA213 ◆
Example 4: Square numbers – expanding a binomial Students apply the distributive law to the expansion of algebraic expressions, including binomials, and collect like terms where appropriate.	ACMNA213 ◆
Example 5: Finding factors – factorising a binomial Students apply the distributive law to the expansion of algebraic expressions, including binomials, and collect like terms where appropriate.	ACMNA213 ◆

Example 1: Reconstruct a rectangle – taking out the common factor

ACMNA213 ◆
Students apply the distributive law to the expansion of algebraic expressions, including binomials, and collect like terms where appropriate.

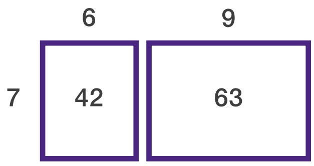
Questions from the BitL tool
Understanding proficiency:
What patterns/connections/ relationships can you see?
Can you represent/calculate in different ways?
Reasoning proficiency:
In what ways can your thinking be generalised?
What can you infer?

Instead of ***telling*** students about taking out a common factor, we can challenge students to recognise the relationships between numbers and expressions for themselves, by ***asking*** questions.

Number warm up

Begin this activity by considering two arrays: 6 x 7 and 3 x 21. Explain that these two arrays cannot be placed together, unchanged, to form a bigger single rectangular array, then ask:

- *How might the 63 points in the 3 x 21 array be rearranged into a different rectangular array?* ((1 x 63), (3 x 21) and (7 x 9). Some students may benefit from using flip counters placed in arrays. If this is the case, choose an example with smaller numbers. Smaller numbers can provide the same challenges and support students with limited number facts to gain conceptual understanding of factors and factorisation.)
- *Is it possible to combine one of the possible arrays of 63 points with the 6 x 7 array to form a bigger single rectangular array?* (Combine (7 x 6) and (7 x 9) into (7 x 15). See Figure 1):



7 x 6 + 7 x 9 = 7 x (6 + 9)
= 7 x 15
= 105

Figure 1

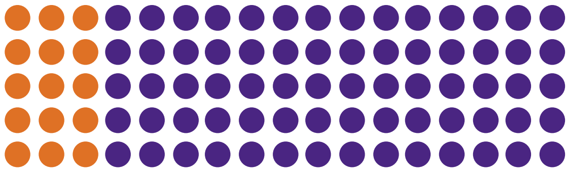
This challenge relates to the factors of the numbers.

Consider the prime factorisation of 42 and 63:
42 = 2 x 3 x 7 63 = 3 x 3 x 7
So: 7 x 6 + 7 x 9 = 7 x (6 + 9)
Alternatively, combine (2 x 21) and (3 x 21) into (5 x 21). (5 x 21) is the array with the longest possible side (besides the trivial case 1 x 105).

- *By looking at the factors of the numbers, how might you have known that 21 is the longest possible non-trivial side?* (21 is the HCF of 42 (2 x 3 x 7) and 105 (3 x 3 x 7)).

Using this thinking, rearrange two arrays (3 x 5) and (2 x 35) so they can be placed in a single rectangular array with the longest possible non-trivial side (see Figure 2):

15 = 3 x **5** 70 = 2 x 7 x **5**



HCF = **5**
(**5** x 3) and (**5** x 14) into (**5** x 17)
ie **5** x (3 + 14)

Figure 2

Challenge students to write their own arrays, possibly requiring three or more arrays to be combined.

A challenge is for students to make up two, three or more arrays that cannot be combined into a single array (other than the trivial case (1 x n)). These numbers are said to be co-prime, for example, 12 (2 x 2 x 3) and 35 (5 x 7).

If students understand prime factorisation but require some support with their number facts, online tools such as the one available on 'Maths is Fun' can be used. 'Maths is Fun' can be found here: http://www.mathsisfun.com/prime-factorization-tool.php

Algebraic expressions

Consider two numbers and their prime factorisations:

$\mathbf{2xy} = 2 \times x \times y$ and $\mathbf{6xz} = 2 \times 3 \times x \times z$

Using the thinking from the *Number warm up*, explain this student's thinking in Figure 3:

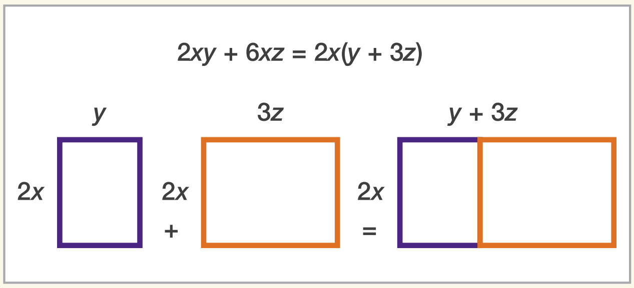


Figure 3

Ask students:

- *What was the HCF of 2xy and 6xz?* (2*x*).
- *Why might mathematicians call this process, 'factorising an algebraic expression by taking out a common factor'?* (The algebraic expression **2*xy* + 6*xz*** changes form from the sum of two terms, to an expression with two factors: **2*x*** *and* **(*y* + 3*z*)**. And so it is said to have been **factor**-ised.)

Set a challenge for students to factorise an algebraic expression by taking out the highest common factor.

(A similar activity also appears in the *Patterns and algebra: Year 10/10A* narrative.)

Example 2: Prime factorisation products – index laws

ACMNA212 ◆
Students extend and apply the index laws to variables, using positive integer indices and the zero index.

Questions from the BitL tool
Understanding proficiency:
What patterns/connections/relationships can you see?
Can you represent/calculate in different ways?
Reasoning proficiency:
In what ways can your thinking be generalised?
What can you infer?

Instead of ***telling*** students to apply index laws, we can challenge students to recognise that they can be used to simplify algebraic products and quotients for themselves, by ***asking*** questions.

In the *Real numbers: Year 9* narrative, 'Example 9: Prime factorisation, index laws and number theory' introduces students to how prime factorisation can be used to understand properties of some very large numbers. Prime numbers are very important for the encryption necessary for security on the internet.

In this activity, learners can build on this knowledge to generalise number patterns. This, in turn, will help them to simplify algebraic expressions involving powers.

Number warm up

Ask students to work collaboratively to find a value for **$(2^3 \times 5^2 \times 11) \times (2^2 \times 5^2)$** in as any ways as they can:

$2^5 \times 5^4 \times 11 = (2^4 \times 5^4) \times 2 \times 11 = 10^4 \times 22 = 220{,}000$

$(8 \times 25 \times 11) \times (4 \times 25) = 200 \times 11 \times 100 = 220{,}000$

Using the calculator:
$2 \times 2 \times 2 \times 5 \times 5 \times 11 \times 2 \times 2 \times 5 \times 5 = 220{,}000$

Card match

For this activity, calculators are not to be used unless as a prompt to support students. Use a card set which contains numerical index products and matching descriptions of the number:

$(2^3 \times 3) \times (2^3 \times 5 \times 7)$	This number ends in one zero and has a factor of 21.
$(2 \times 7^2) \times (3^3 \times 5)$	This number is even, but the number that's half its value is odd.
$(2^3 \times 5^3 \times 7) \times (2 \times 5 \times 7)$	This number is a perfect square. The square root of the number is $2^2 \times 5^2 \times 7$.
$(3^2 \times 5) \times (7^5 \times 5)$	This number is odd and has factors of 25 and 9.
$(2^4 \times 5^7) \times (5^3 \times 2^6)$	This number only has digits which are either 0 or 1.

The task can be differentiated by adding pronumerals where there may be more than one answer depending on the value of the pronumeral (all pronumerals are prime):

$(2^5 \times a) \times (b^3 \times 5^2)$	This number ends in two zeros, has a factor of 3 and at least one factor of 7 (for this match **a** = 3 and **b** = 7).
$(2 \times 3^2) \times (m^3 \times n)$	This number has only one factor of 6 and ends in one zero (for this match neither **m** nor **n** can equal 3. Either **m**, **n** or both must be 5).
$(3^2 \times 5^4) \times p$	This number is odd and ends in a 5 (for this match, **p** cannot be 2).
$(t^2 \times 7^2) \times 5^2$	This number is even, has a factor of 49 and ends in 2 zeros (for this match, **t** must be 2).

An extension for any game or card sort, is for students to make up their own.

Ask students to use the skills they used to simplify the products, to suggest how this algebraic product could be simplified when the numbers are unknown and represented by pronumerals:

$(2 \times a^3 \times b) \times (a^4 \times 5 \times b^2)$

Students learn by noticing: Invite their curiosity

Rather than explaining index laws and prime factorisation, students can demonstrate their understanding by sorting the cards and asking students what they are learning from it.

Ask groups to present their thinking to the class:

- *How might we check your ideas?* (The expressions could be verified by substituting in values for the pronumeral.)

Other activities to be used as an extension:

- Present four different algebraic products and ask students to sort them from easiest to hardest.
- Ask students to give feedback to other students' misconceptions.
- Repeat this process with a numerical introduction for algebraic quotients with index laws.

Example 3: One above/one below – difference of two squares

ACMNA213 ◆
Students apply the distributive law to the expansion of algebraic expressions, including binomials, and collect like terms where appropriate.

Questions from the BitL tool

Understanding proficiency:

What patterns/connections/relationships can you see?

Can you represent/calculate in different ways?

Reasoning proficiency:

In what ways can your thinking be generalised?

What can you infer?

Instead of ***telling*** students about the rules for binomial expansions, we can challenge students to recognise the relationships for themselves, by using manipulatives and ***asking*** questions.

Begin this activity by choosing any number (for example, **5**) and square it (**25**). Pick the number that is one below 5 (**4**) and the number that is one above 5 (**6**). Find their product (**24**).

Notice that the result is one less than **5²**. Is this always true?

Instruct students to try this using small and large numbers (including fractions), then ask:

Number	One less	One more	Product	Square
8	7	9	63	64
50	49	51	2499	2500
1.5	0.5	2.5	1.25	2.25

- *Why does it work? How might you demonstrate this with the two coloured flip tiles?* (Starting with 5^2, make one side one less and use these tiles to make the other side one more. You always have one left over so $5^2 - 1 = (5 - 1) \times (5 + 1)$. See Figure 4.)

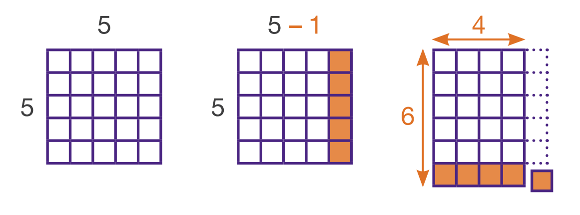


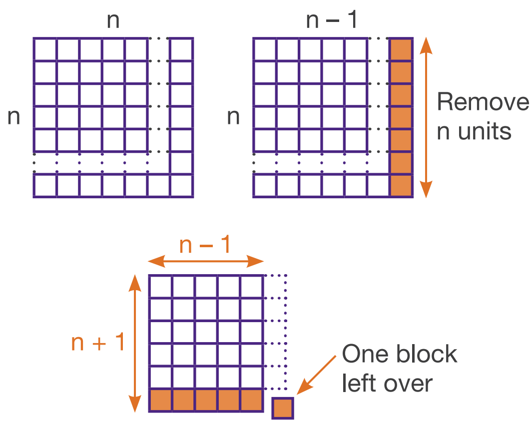
Figure 4

- *When would this be useful?* (Consider 99^2. One less than 99 is 98. One more than 99 is 100. The product, 98 x 100 = 9800, is one less than the square of 99, so 99^2 must be one more (ie 9800 + 1 = **9801**). 101^2 can also be calculated this way.)

- *Can you generalise?* (Remember to ask students to explain the pattern in words before they attempt the abstract algebraic expression. See Figure 5.)

$(n + 1)(n - 1) = n^2 - 1$

Product of (one up) and (one down) is one less than the square of the number.



Need to add an extra row of (n – 1) blocks.

Figure 5

(This activity also appears in the Patterns and algebra: Year 10/10A narrative.)

Example 4: Square numbers – expanding a binomial

ACMNA213 ◆
Students apply the distributive law to the expansion of algebraic expressions, including binomials, and collect like terms where appropriate.

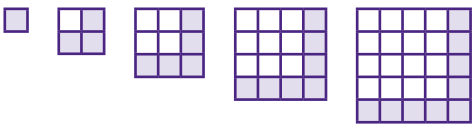
Questions from the BitL tool
Understanding proficiency:
What patterns/connections/relationships can you see?
Can you represent/calculate in different ways?
Reasoning proficiency:
In what ways can your thinking be generalised?
What can you infer?

Instead of ***telling*** students about the rules for binomial expansions, we can challenge students to recognise the relationships for themselves, by using manipulatives and ***asking*** questions.

Begin this activity by discussing the following with your class:

- *What might the next 4 numbers in the sequence 1,4,9, … be? What pattern did you notice?* (The numbers are perfect squares, but some students may notice that the numbers differ by consecutive odd numbers.)
- *Would this always be the case for perfect squares? Can you convince me?* (No matter how many numbers you check, you cannot **prove** that it would always be the case. You are only verifying the conjecture you have made. Challenge students to make a convincing argument using manipulatives. See Figure 6.)

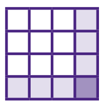
Using flip tiles, construct the next square number from the previous one.



Consider why the construction would always require an odd number of tiles.

Figure 6

- *Would the construction always require an odd number of tiles? Can you convince me?* (To make the sixth square you need to add 5 to the right-hand side and 5 to the bottom side of the fifth square. This is an even number because it is 2 lots of 5, but then you need to put one more square in the bottom right-hand corner, so one added to an even number is always odd (ie 2 x 5 + 1 = 11).)
- *Can you generalise?* (In the general case, for a (n x n) square, n units are added to the side, the bottom and one on the corner and this made:
(n+1) x (n+1)
$n^2 + 2 \times n + 1 = (n+1)^2$
Explain how Figure 7 below shows that the number you add to get from one square number to the next, is always 'double the number plus 1'.)



For a 3 x 3 square, 3 units are added to the side, the bottom and one for the corner and this made a 4 x 4:

$3^2 + (2 \times 3 + 1) = 16 = 4^2$

Figure 7

- *Can you prove this another way?* (Students may use an area model as in 'Example 3: One above/one below – difference of two squares'.)

(This activity also appears in the <u>*Patterns and algebra: Year 10/10A*</u> narrative.)

Example 5: Finding factors – factorising a binomial

ACMNA213 ◆
Students apply the distributive law to the expansion of algebraic expressions, including binomials, and collect like terms where appropriate.

Questions from the BitL tool

Understanding proficiency:
What patterns/connections/relationships can you see?
Can you represent/calculate in different ways?

Reasoning proficiency:
In what ways can your thinking be generalised?
What can you infer?

Instead of ***telling*** students about the rules for binomial expansions, we can challenge students to recognise the relationships for themselves, by using interactives and ***asking*** questions.

This **Finding factors** activity from the NRICH website is a problem-solving task that provides practice at factorising monic quadratic expressions. Doing **Missing Multipliers** first (available at: **http://nrich.maths.org/7382&part**) will familiarise students with both the interactive and the challenge; as well as reminding them that algebraic expressions are merely a generalised form of numbers, which they already know a lot about.

The link to the problem on the NRICH website is: http://nrich.maths.org/7452

(This activity also appears in the *Patterns and algebra: Year 10/10A* narrative.)

Proficiency: Problem-solving

Proficiency emphasis and what questions to ask to activate it in your students (Examples 6–9)

Problems are described in the AC: Mathematics as 'meaningful' or 'unfamiliar'. Students of *all* abilities and ages should be provided with experiences of both meaningful and unfamiliar problems.

Meaningful problems

Meaningful problems are set in a context that a student can project themselves into. It may be that the mathematics and strategy being applied is familiar to the student or the problem relates to their own life experience. Connecting with a context does not mean that the students have to see it as 'fun' nor does it have to relate to an immediately practical situation from daily life.

Students can connect with a problem through provocations such as the use of manipulatives (either physical or digital) or through a dramatisation (eg, a story, interesting background information, a video clip). The intention is to give students the opportunity to work as a mathematician would work, in a context that they can access at their current stage of development.

Unfamiliar problems

Fundamentally there are two groups of unfamiliar problems:

- Problems for which the students would not be able to say that they had done a similar example previously, they would therefore need to create an approach (develop a strategy).
- Problems in which the students develop a new piece of knowledge. They begin the problem by applying the knowledge/skills that they have and they complete the problem having recombined that knowledge to form a new piece of understanding.

Growth mindset: Learning that not knowing is the beginning of a learning opportunity

Unfamiliar problems tend to provoke a response of, 'I don't know', or 'I'm not sure'. Students respond differently to this feeling; some shut down, others begin to ask, 'But how could I work that out?'

In developing powerful learners we are aiming for all of our students to learn that 'not knowing' is the *beginning of a learning opportunity* and that the first move that they need to make on the journey to finding out more is to ask, 'What could I do to work this out?'

Engaging in problem-solving supports the move *from tell to ask*

Instead of ***telling*** students:

- the problem to solve
- the information they'll need
- the steps they should take.

We can ***ask students to identify***:

- the problem to solve
- the information they'll need
- a possible process to use.

Proficiency: Problem-solving examples	
Example 6: 2-digit square – expanding quadratics Students apply the distributive law to the expansion of algebraic expressions, including binomials, and collect like terms where appropriate.	ACMNA213 ◆
Example 7: Quadratic patterns Students apply the distributive law to the expansion of algebraic expressions, including binomials, and collect like terms where appropriate.	ACMNA213 ◆
Example 8: Hollow squares – factorisation Students apply the distributive law to the expansion of algebraic expressions, including binomials, and collect like terms where appropriate.	ACMNA213 ◆
Example 9: What's possible? – difference of squares Students apply the distributive law to the expansion of algebraic expressions, including binomials, and collect like terms where appropriate.	ACMNA213 ◆

Example 6: 2-digit squares – expanding quadratics

ACMNA213 ◆
Students apply the distributive law to the expansion of algebraic expressions, including binomials, and collect like terms where appropriate.

Questions from the BitL tool

Problem-solving proficiency:
Interpret; Model and plan; Solve and check; Reflect.

Reasoning proficiency:
What can you infer?

Instead of ***telling*** students how to expand algebraic products, we can challenge students to recognise the relationships for themselves, by ***asking*** questions.

This task from the NRICH website involves the search for a 2-digit number with a special property. The number can be determined by using the concept of place value and algebraic representations, expanding and factorising.

An extension to this task might be to challenge students to see if there is a 3-digit number with these properties.

The link to the problem on the NRICH website is: http://nrich.maths.org/517

Example 7: Quadratic patterns

ACMNA213 ◆
Students apply the distributive law to the expansion of algebraic expressions, including binomials, and collect like terms where appropriate.

Questions from the BitL tool

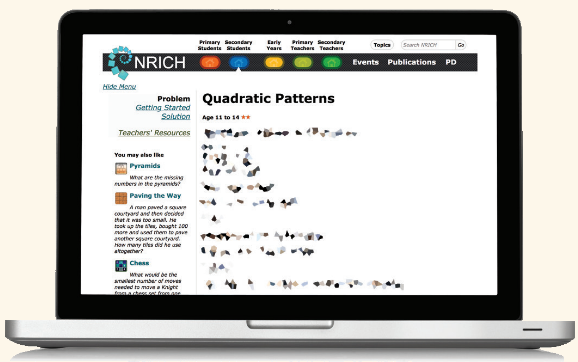
Problem-solving proficiency:
Interpret; Model and plan; Solve and check; Reflect.

Reasoning proficiency:
What can you infer?

Instead of ***telling*** students about number patterns and how they can be represented using algebraic notation, we can challenge students to recognise the relationships for themselves, by ***asking*** questions.

This activity from the NRICH website gets students to problem-solve by connecting number patterns to the algebraic and area representations.

The link to the problem on the NRICH website is: http://nrich.maths.org/11011



Example 8: Hollow squares – factorisation

ACMNA213 ◆
Students apply the distributive law to the expansion of algebraic expressions, including binomials, and collect like terms where appropriate.

Questions from the BitL tool

Problem-solving proficiency:
Interpret; Model and plan; Solve and check; Reflect.

Reasoning proficiency:
What can you infer?

Instead of ***telling*** students how to factorise a quadratic expression, we can challenge students to recognise the relationships for themselves, by ***asking*** questions.

In the form of a backwards question, this investigation from the NRICH website explores the 'hollow square' infantry formation in the Napoleonic battles, for differing numbers of soldiers.

The link to the problem on the NRICH website is: http://nrich.maths.org/11257

Example 9: What's possible? – difference of squares

ACMNA213 ◆
Students apply the distributive law to the expansion of algebraic expressions, including binomials, and collect like terms where appropriate.

Questions from the BitL tool

Problem-solving proficiency:
Interpret; Model and plan; Solve and check; Reflect.

Reasoning proficiency:
What can you infer?

Instead of ***telling*** students how to use algebra for justification and proof, we can challenge students to recognise the relationships for themselves, by ***asking*** questions.

This investigation from the NRICH website explores whether all integers can be written as the difference of two squares.

The link to the problem on the NRICH website is: http://nrich.maths.org/742

Connections between 'Patterns and algebra' and other maths content

There are many opportunities to connect to other content in the AC: Mathematics, when we use patterns and algebra as a starting point.

Here are just some of the possible connections that can be made:

Mathematics: Year 9	
Whilst working with Patterns and algebra, connections can be made to:	**How the connection might be made:**
Students apply index laws to numerical expressions with integer indices. ACMNA209	Refer to: **Example 2: Prime factorisation products – index laws**

Making connections to other learning areas

We know that when our students meet a concept frequently and in different contexts, they have a greater chance of developing understanding. With this in mind, it is our responsibility help our students to make these connections by intentionally designing tasks that connect a number of different content descriptions. Alternatively, connections can be made through questioning individual or small groups of students.

‘Patterns and algebra’ from Foundation to Year 10A

The AC: Mathematics year level content descriptions shown here have been colour coded to highlight the following curriculum aspects of working with Patterns and algebra:

Copy, continue and create patterns ◆
In **Foundation** through the ‘patterns and algebra content descriptions’, students copy, continue and create patterns.

Investigate and describe number patterns ◆
In **Years 1 to 6** students are expected to investigate and describe number patterns.

Use variables to represent numbers and create algebraic expressions ◆
In **Year 7** students mostly use variables to represent numbers and create algebraic expressions.

Simplify and identify equivalent algebraic expressions by extending and applying laws and properties of numbers ◆
In **Year 8 and Year 10** students mostly simplify and identify equivalent algebraic expressions by extending and applying laws and properties of numbers.

Use algebraic thinking and processes to solve problems ◆
In **Years 3 to 6**, students use algebraic thinking to solve problems. In **Year 10/10A** students mostly use algebraic thinking and processes to solve problems.

Year level	‘Patterns and algebra’ content descriptions from the AC: Mathematics
Foundation ◆	Students sort and classify familiar objects and explain the basis for these classifications. Students copy, continue and create patterns with objects and drawings. ACMNA005
Year 1 ◆	Students investigate and describe number patterns formed by skip-counting and patterns with objects. ACMNA018
Year 2 ◆ ◆	Students describe patterns with numbers and identify missing elements. ACMNA035
Year 2 ◆	Students solve problems by using number sentences for addition or subtraction. ACMNA036
Year 3 ◆	Students describe, continue, and create number patterns resulting from performing addition or subtraction. ACMNA060
Year 4 ◆	Students explore and describe number patterns resulting from performing multiplication. ACMNA081
Year 4 ◆	Students solve word problems by using number sentences involving multiplication or division where there is no remainder. ACMNA082
Year 4 ◆	Students find unknown quantities in number sentences involving addition and subtraction and identify equivalent number sentences involving addition and subtraction. ACMNA083
Year 5 ◆	Students describe, continue and create patterns with fractions, decimals and whole numbers resulting from addition and subtraction. ACMNA107
Year 5 ◆	Students find unknown quantities in number sentences involving multiplication and division and identify equivalent number sequences involving multiplication and division. ACMNA121
Year 6 ◆ ◆	Students continue and create sequences involving whole numbers, fractions and decimals. Students describe the rule used to create the sequence. ACMNA133
Year 6 ◆	Students explore the use of brackets and order of operations to write number sentences. ACMNA134
Year 7 ◆	Students introduce the concept of variables as a way of representing numbers using letters. ACMNA175

Year 7 ◆	Students create algebraic expressions and evaluate them by substituting a given value for each variable. ACMNA176
Year 7 ◆	Students extend and apply the laws and properties of arithmetic to algebraic terms and expressions. ACMNA177
Year 8 ◆	Students extend and apply the distributive law to the expansion of algebraic expressions. ACMNA190
Year 8 ◆	Students factorise algebraic expressions by identifying numerical factors. ACMNA191
Year 8 ◆	Students simplify algebraic expressions involving the four operations. ACMNA192
Year 9 ◆	Students extend and apply the index laws to variables, using positive integer indices and the zero index. ACMNA212
Year 9 ◆	Students apply the distributive law to the expansion of algebraic expressions, including binomials, and collect like terms where appropriate. ACMNA213
Year 10 ◆	Students factorise algebraic expressions by taking out a common algebraic factor. ACMNA230
Year 10 ◆	Students simplify algebraic products and quotients using index laws. ACMNA231
Year 10 ◆	Students apply the four operations to simple algebraic fractions with numerical denominators. ACMNA232
Year 10 ◆	Students expand binomial products and factorise monic quadratic expressions using a variety of strategies. ACMNA233
Year 10 ◆	Students substitute values into formulas to determine an unknown. ACMNA234
Year 10A ◆	Students investigate the concept of a polynomial and apply the factor and remainder theorems to solve problems. ACMNA266

Numeracy continuum: Recognise and use patterns and relationships	
End Foundation	Describe and continue patterns.
End Year 2	Identify, describe and create everyday patterns.
End Year 4	Identify and describe trends in everyday patterns.
End Year 6	Identify and describe pattern rules and relationships that help to identify trends.
End Year 8	Identify trends using number rules and relationships.
End Year 10	Explain how the practical application of patterns can be used to identify trends.

Source: ACARA, Australian Curriculum: Mathematics

Resources

NRICH website

http://nrich.maths.org

In this conceptual narrative we have highlighted the possibility of using tasks from an organisation called NRICH enriching mathematics.

The NRICH website contains a large collection of high quality maths problem-solving tasks, together with suggestions about content that may be related to the task, ways to get started and different (valid) solutions that have been submitted by students from around the world.

Dan Meyer's blog: 101 questions

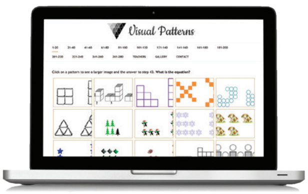
http://www.101qs.com

Dan's blog contains images and short films that can be presented to students along with the question: *What's the first question that comes to mind?*

A spreadsheet of ***Dan Meyer's Three-Act Maths Tasks*** can be accessed at http://bit.ly/DM3ActMathTasks.

Visual patterns

http://www.visualpatterns.org/



This website has multiple visual patterns that students can consider and describe in a way that makes sense to them. Students will have multiple interpretations of each image, promoting multiple ways of visualising, solving problems and stimulating dialogue. They can be used as lesson starters.

Scootle

https://www.scootle.edu.au/ec/p/home

This website has over 20,000 quality-assured digital learning resources aligned to the Australian Curriculum. You can filter your search to uncover a wealth of relevant teaching and learning items.

reSolve: maths by inquiry

https://www.resolve.edu.au

This website provides classroom resources for years F to 10 that promote fluency, deep understanding, strategic problem-solving, and mathematical reasoning. Each classroom resource is designed to develop progressive understanding through tasks that encourage a spirit of inquiry.

Plus Magazine

https://plus.maths.org

An online magazine which aims to introduce readers to the beauty and the practical applications of mathematics. It includes articles, teaching packages, puzzles and more. It has examples of mathematical modelling from scaffolded to open in authentic contexts.

Numeracy in the News

http://www.mercurynie.com.au/mathguys/mercury.htm

Numeracy in the News is a website containing 313 full-text newspaper articles from the Tasmanian paper, *The Mercury*. Other News Limited newspapers from around Australia are also available. The intention of the website is to raise the consciousness of students as critical readers of media reports, including statistical inference. The 'Teacher discussion' notes are a great example of how you can adapt student questions to suit articles from our local papers, such as *The Advertiser*.

TIMES modules

http://schools.amsi.org.au/times-modules/

TIMES modules are prepared by the Australian Mathematical Sciences Institute (AMSI) as part of The Improving Mathematics Education in Schools (TIMES) Project. The 'Data investigation and interpretation' module has been developed for Foundation to Year 10, and is a great knowledge source for teachers, modelling good analysis and inference reports.

Top drawer teachers – resources for teachers of mathematics (statistics)

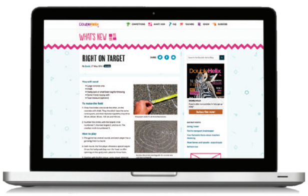
http://topdrawer.aamt.edu.au/Statistics

This website by the Australian Association of Mathematics Teachers, provides expert mathematics advice, teaching suggestions and classroom activities. Each 'drawer' is divided into sections: Big ideas, Misunderstandings, Good teaching, Assessment, and Activities.

Double Helix Extra

https://blog.doublehelix.csiro.au/

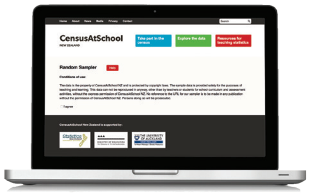
This CSIRO Double Helix Extra is a free fortnightly email newsletter featuring mathematics news and activities. It includes a quiz, brainteaser, news and a classic hands-on activity.



CensusAtSchool NZ

http://new.censusatschool.org.nz/tools/random-sampler/

CensusAtSchool NZ is part of a global project that provides a random sampler and additional resources for teaching statistics.



It aims to:

- *'foster a positive attitude to statistics through using data that is both relevant and real*
- *improve understanding of a data gathering process, its purposes and benefits to society*
- *provide access to large and meaningful multivariate data sets*
- *encourage effective IT teaching and learning*
- *enhance the process of statistical enquiry across the curriculum.'*

Notes

Do you want to feel more confident about the maths you are teaching?

Do you want activities that support you to embed the proficiencies?

Do you want your students thinking mathematically rather than just doing maths?

If you answered **yes** to any of these questions, then this resource is for you.

Packed full of examples, along with questions you can ask students as they engage in their learning, this resource supports you to develop confidence in teaching the Australian Curriculum: Mathematics.

Produced by the Learning Improvement division

Excluded from NEALS