

Using units of measurement: Years 5–7

MATHEMATICS CONCEPTUAL NARRATIVE

Leading Learning: Making the Australian Curriculum work for us
by bringing CONTENT and PROFICIENCIES together



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Resource key



This teacher will raise questions, answer students’ questions and share some of her classroom practice.



This teacher will give you his top pedagogy tips.



These students will raise questions and model student thinking.

Bringing it to Life (BitL): key questions are in bold orange text.

Sub-questions from the BitL tool are in green medium italics – these questions are for teachers to use directly with students.

What the Australian Curriculum says about ‘using units of measurement’

Content descriptions

Strand | Measurement and geometry

Sub-strand | Using units of measurement

Year 5 | ACMMG108 | Students choose appropriate units of measurement for length, area, volume, capacity and mass.

Year 5 | ACMMG109 | Students calculate the perimeter and area of rectangles using familiar metric units.

Year 6 | ACMMG135 | Students connect decimal representations to the metric system.

Year 6 | ACMMG136 | Students convert between common metric units of length, mass and capacity.

Year 6 | ACMMG137 | Students solve problems involving the comparison of lengths and areas using appropriate units.

Year 6 | ACMMG138 | Students connect volume and capacity and their units of measurement.

Year 7 | ACMMG159 | Students establish the formulas for areas of rectangles, triangles and parallelograms and use these in problem solving.

Year 7 | ACMMG160 | Students calculate volumes of rectangular prisms.

Year level descriptions

Year 5 | Students choose appropriate units of measurement for calculation of perimeter and area.

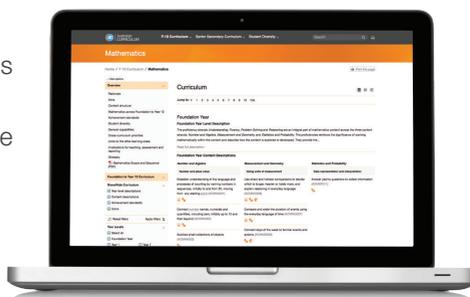
Year 5 | Students formulate and solve authentic problems using whole numbers and measurements.

Year 6 | Students measure using metric units.

Year 6 | Students formulate and solve authentic problems using fractions, decimals, percentages and measurements.

Year 7 | Students calculate areas of shapes and volumes of prisms.

Year 7 | Students formulate and solve authentic problems using numbers and measurements.



Source: ACARA, Australian Curriculum: Mathematics, Version 8.0

Achievement standards Numeracy continuum

Year 5 | Students use appropriate units of measurement for length, area, volume, capacity and mass, and calculate perimeter and area of rectangles.

Year 6 | Students connect decimal representations to the metric system and choose appropriate units of measurement to perform a calculation. They make connections between capacity and volume. They solve problems involving length and area.

Year 7 | Students use formulas for the area and perimeter of rectangles and calculate volumes of rectangular prisms.

End of Year 6 | Estimate and measure with metric units

Students choose and use appropriate metric units for length, area, volume, capacity and mass to solve everyday problems.

End of Year 8 | Estimate and measure with metric units

Students convert between common metric units for volume and capacity and use perimeter, area and volume formulas to solve authentic problems.

In the Australian Curriculum: Mathematics, the concept of 'time' is addressed in the sub-strand 'Using units of measurement', but in this resource, 'time' has its own narrative.

Working with units of measurement

Important things to notice

When we design learning about measurement, it is easy to think solely about 'using measuring instruments'. It is also important to design opportunities for students to:

- **select** appropriate units
- **identify** measurable attributes
- **develop language** associated with measurement
- **estimate** using metric units.



I wonder if all standard units are metric?



I want to become great at estimating measurements!



I need to be challenged to choose appropriate units myself.



There is lots of measurement language that I'll need time to practise using.

Building on learning from past years

A quick look at Year 3 to Year 4

In Year 3, students make the transition from the use of informal units to familiar metric units. During Year 3 familiar metric units are introduced for length, mass and capacity. Working with metric units is extended in Year 4 to include familiar metric units for area and volume.

Familiar metric units are metric units that would most commonly be experienced by a student of that age. For example, centimetres, metres, grams and kilograms would be more familiar (and relevant) to young learners than kilometres, millimetres, tonnes and milligrams.

Developing an ability to estimate

Different ways to reason when estimating length

Is estimating guessing?



No. Estimating is reasoning, not guessing. We can estimate in lots of different ways.



Typically we estimate distance in one of the following ways:

A comparison to another known distance: Pacing or marking out:

I know my classroom is about 5m wide and this room is about the same as my classroom.



I know my paces are about half a metre, so 10 paces is 5 metres.



A comparison to another length and then an adjustment:

I am about 150 cm tall and this cupboard is a bit taller than me, so it's about...



Visualise/mark out the unit distance and count:

I can think about how long a 'one metre' ruler is and visualise laying 'one metre' rulers end to end across the room. Sometimes, I visualise the rulers to the half way point and double that measurement.



Engaging learners

Harnessing students' fascination with scale

People are often fascinated with very large or very small items. We are particularly fascinated with large items that should be small and small items that should be large. For an example of this fascination, follow the link below to the news story about giant marionettes in Perth, WA. An estimated 1.4 million people attended 'The Giants extravaganza'!

There are films, such as 'The Borrowers' and 'Gulliver's Travels', that play on our fascination with scale. Such films and images can be used to make connections between measurement, scale, enlargement and fractions.

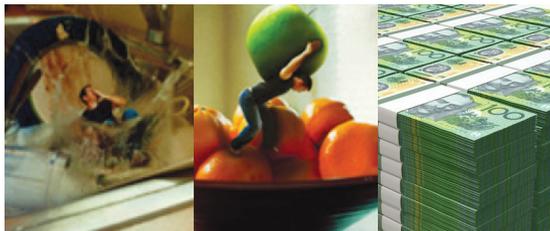


<http://www.perthnow.com.au/news/western-australia/giants-in-perth-day-three/story-fnhocx03-1227220081679>

Perth Now
February 2015
Picture: Stewart Allen

To use this story to engage learners we would play one of the news stories, without the audio (at first) and ask students:

What questions do you have?



Images of large amounts of money and movie scenes that involve the transaction of large amounts of cash in small bags, or briefcases, provide another engaging context for working with units of measurement.

We can support our students to develop a disposition towards using maths in their lives, ie becoming numerate, not only through the use of 'real world' maths problems, but through fostering a disposition towards asking mathematical questions about everything they see. We develop this disposition in our students when we promote, value and share their curiosity and provide opportunities for them to develop their questions and explore solutions to their questions.

Embedding the Australian Curriculum: Mathematics proficiencies

Pedagogy supporting you to embed the proficiencies

AC: Mathematics proficiencies

The verbs used in the four Mathematics proficiencies from the Australian Curriculum (AC: Proficiencies) describe the actions in which students can engage when learning and using mathematics content.

To embed the AC: Proficiencies in students learning experiences, we need to ask questions that activate those actions in students. But what questions will achieve this? The AC: Proficiencies describe the actions, but not the questions that can drive those actions.



There are four proficiency strands in the Australian Curriculum: Mathematics:

- Understanding
- Problem Solving
- Reasoning
- Fluency.



Bringing it to Life tool

The Bringing it to Life (BitL) tool was developed by the South Australian Teaching for Effective Learning (TfEL) team, to support teachers to bring the AC: Proficiencies to life in the classroom. The BitL tool models questions that can be used to drive the actions described in the AC: Proficiencies.



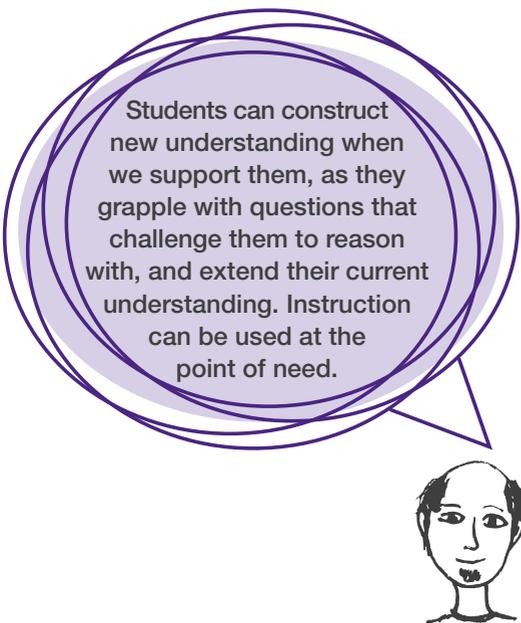
The Bringing it to Life tool is located in the *Leading Learning: Making the Australian Curriculum Work for Us* resource (www.acleadersresource.sa.edu.au), in the section *Bringing it to Life – essence meets content*.



Beware of the old paradigm

There is a prevalent assumption that we should instruct our students with processes and develop their understanding before we challenge them to problem solve and reason. In this paradigm students will only gain problem solving experience at the end of the unit of work, assuming they get through all of the practice questions quickly enough.

The pedagogy shift of innovative educators across the world acknowledges that new understanding and the ensuing fluency are not simply a resource for problem solving and reasoning, but a product of problem solving and reasoning.



Students can construct new understanding when we support them, as they grapple with questions that challenge them to reason with, and extend their current understanding. Instruction can be used at the point of need.

Why does this resource look at each proficiency separately, when they are intertwined skills?

We acknowledge that the proficiencies intertwine and that it is possible to experience a range of proficiencies within one particular problem. However, we have used the BitL questions to organise the examples into categories that emphasise each particular proficiency. The intention in doing this is to support teachers to understand the emphasis of each proficiency deeply in order to be able to intertwine them as appropriate.

You will find less of an emphasis, in this resource on fluency examples, as many textbook and worksheet resources already provide this.

Examples modelling embedding the proficiencies using BitL questions:

- **Understanding**
Examples 1–11
- **Problem Solving**
Examples 12–14
- **Reasoning**
Examples 15–16
- **Fluency**
Examples 17–24

It is intended that teachers will select/adapt and sequence examples that are appropriate for their students. These examples have been grouped by proficiency, not learning sequence.

Proficiency: Understanding

Proficiency emphasis and what questions to ask to activate it in your students (Examples 1–11)

The AC: Mathematics defines the proficiency of ‘Understanding’ as:

What the
AC says



Proficiency: Understanding

Students build a robust knowledge of adaptable and transferable mathematical concepts. They make connections between related concepts and progressively apply the familiar to develop new ideas. They develop an understanding of the relationship between the ‘why’ and the ‘how’ of mathematics. Students build understanding when they connect related ideas, when they represent concepts in different ways, when they identify commonalities and differences between aspects of content, when they describe their thinking mathematically and when they interpret mathematical information.

BitL tool



There are three BitL questions associated with this proficiency. They reflect the student actions as described in the AC: Mathematics. The three questions are:

Q1 What patterns/connections/relationships can you see?

Q2 Can you answer backwards questions?

Q3 Can you represent or calculate in different ways?

Q1

Examples
overview



What patterns/connections/relationships can you see?

The intent of this question is to promote learning design that intentionally plans for students to develop a disposition towards looking for patterns, connections and relationships.

Example 1: Area of a rectangle

Establishing formulae for calculating the area of a rectangle

Example 2: Area of a triangle

Establishing formulae for calculating the area of a triangle

Example 3: Area of a parallelogram

Establishing formulae

Example 4: Converting between units

Length, mass and capacity—connecting decimals to the metric system

Q1 Examples overview

Example 5: Connecting volume and capacity

Connections between units of different attributes (volume and capacity)

Example 6: Relationships between attributes

Area and perimeter

Q2 Examples overview



Can you answer backwards questions?

The intent of this question is to promote learning design that intentionally plans for students to develop flexibility in the way that they can work with a concept.

Example 7: Rectangles—connecting area and perimeter

Applying formulae for calculating the area of a rectangle

Example 8: Volume of a rectangular prism

Applying formulae for calculating the volume of a rectangular prism

Example 9: Area of triangles and parallelograms

Applying formulae for calculating the area of triangles and parallelograms

Example 10: Calculations using the four operations in the context of measurement

Q3 Examples overview



Can you represent or calculate in different ways?

The intent of this question is to promote learning design through which students experience multiple representations and create multiple approaches. We encourage teachers to look for opportunities to:

- present information/problems in a range of ways
- ask the questions:

Is there another possibility?

Is there another way?

Example 1, Example 6, Example 7 and Example 8 all require students to think about multiple ways to create, represent or calculate in relation to area.

Example 11: Ordering in different ways

Using different attributes

Example 1: Area of a rectangle

Establishing formulae for calculating the area of a rectangle

Encourage students to record length, width and area of the rectangles in a table like the one in Figure 1, and ask:

What connections do you notice?



Figure 1

Providing (or asking students to create) a group of rectangles that all have the same area can help to avoid the distraction of all values being different.

We can challenge students to generalise their observation and create a rule for calculating the area of a rectangle.

Students who can already generalise the rule for calculating the area of a rectangle can begin to explore rules for other 2D shapes. The same process can be used with older students for calculating the volume of a rectangular prism.



Often, it can be easier to notice a pattern or connection when data is arranged in a table. Arranging the data in a logical/progressive order is also beneficial on many occasions and at least not detrimental on any occasion, so the use of tables and logical/progressive arrangements are good strategies to model with students.

LEARNING IN CONTEXT

When we provide the conditions for students to spot connections for themselves, often we remove the need to 'tell' students facts and instruct processes. This mode of learning is empowering for learners. Knowing they have the capacity to construct their own knowledge and understanding, rather than simply receive and use knowledge, underpins being a powerful learner.



Example 2: Area of a triangle

Establishing formulae for calculating the area of a triangle

First ask students about Figure 2:

What's the same about the way that each of the triangles has been formed here?

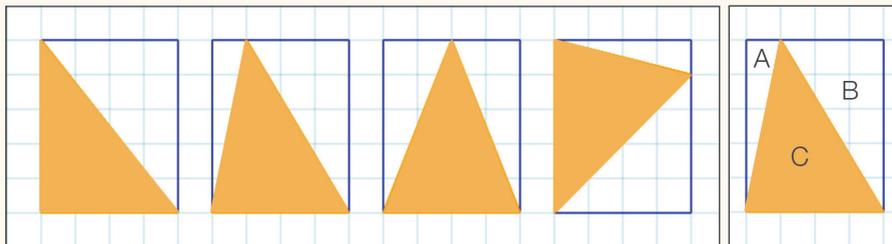


Figure 2

Figure 3

There are many ways to describe this, but essentially, one edge of the triangle is always the same length as one edge of the rectangle and the highest point of the triangle is as tall as the other edge of the rectangle.

Another way to think about this is that a rectangle has been drawn around the triangle. The specifics of the language don't really matter at this stage. What does matter is that the students see a connection between the rectangle and the triangle.

We can continue to develop student thinking by asking:

Could you generate some more possible triangles that fit inside this rectangle? Show me.

What fraction of the rectangle is covered by the triangle in the first image?

What about the next image and the next etc ...? Convince me/yourself!

It doesn't matter if the student's first idea is right or wrong. If it's wrong, they will find that out and they can try, or be directed towards, a different idea, until they reach the point of understanding that the triangle is half of the area of the rectangle that fits around it. You can see, in Figure 3, that triangles A and B could be cut off, rotated and positioned to cover triangle C.

Support further development of understanding by asking:

*What's the area of the rectangle?
So what's the area of the triangle?*

What's the connection between the area of the rectangle and the area of the triangle?

Encourage students to generalise, by asking:

Is there a rule that you could use to describe a way to work out the area of a triangle?

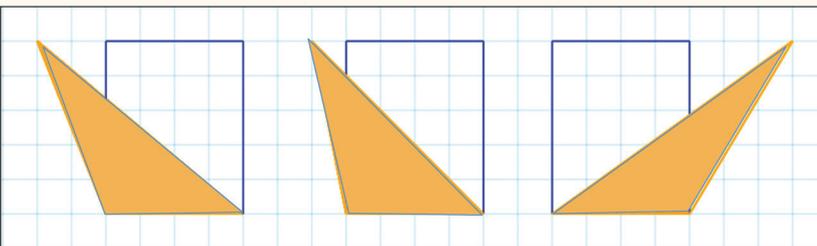


Figure 4

What if you change the size of the rectangle/triangle? Does your rule work for any size of triangle?

What if the triangle is still as tall as the rectangle and it still shares the same base, but it falls outside the rectangle, as in Figure 4. Does your rule still work now? Is the triangle still half the area of the rectangle with the same base and height? Convince yourself. Convince me.

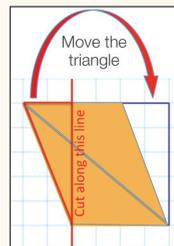


Figure 5

It is very likely that this question will now become a problem solving question for many students. Rearranging this form of triangle to create half of the rectangle is problematic. Most students will try to cut off the section of the triangle that overhangs the rectangle and place it inside the rectangle in the hope of making clear that half of the rectangle is now covered. This tends not to work. We can allow students to discover this for themselves, then challenge them to think about other ways they could show the triangle to be half of the rectangle.

We could use the following enabling prompts:

If the triangle was half the area of the rectangle, how many triangles would it take to cover all of the rectangle? (Two)

Could you try using two triangles to make one rectangle? (Refer to Figure 5 for an example of how this can work)

LEARNING IN CONTEXT

Using variables to establish formulae for the area/volume of shapes is a good context to experiment with variables because it is related to a concrete representation. It is beneficial for students to experiment with different representations of their formula, comparing their formula to their peers and then to conventional representations that they can find on the internet or in textbooks.



Example 2 continued

Language associated with triangles

When working with triangles it is useful to introduce the language 'base' and 'height'. These words are used in the same kind of way as 'length' and 'width' for rectangles and squares. It is important to note that the base and height are at right angles to each other. This will sometimes mean that the height goes through the middle of the triangle, rather than being along one edge.

Rather than telling students what base and height are, in relation to triangles, first find out what they think. We can say:

'Base' and 'height' are words that we (as mathematicians) will use to describe lengths on triangles. Where do you think the base and height would be on this triangle? (Draw a triangle)

Show students images such as those in Figure 6, revealing the images one by one and allowing students to adjust their understanding of the terms base and height.

Notice:

- The base is always one of the sides of the triangle.
- The height is always at right angles to the base.

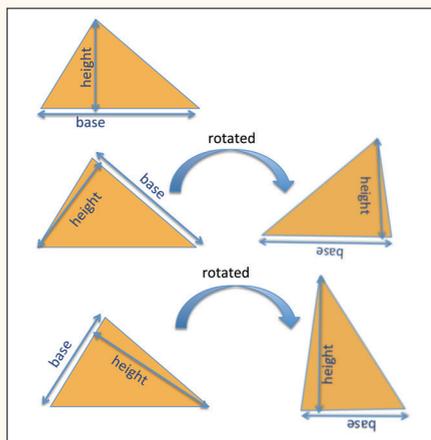


Figure 6

Challenge students thinking by saying:

In these images the height is not one of the sides of the triangle. For what type of triangle could the height be one of the sides of the triangle? (A right angle triangle)

The key point to establish is that base and height are relative to each other. In maths worksheets the base is often presented as a horizontal side on a triangle. This causes confusion for students when they are presented with a triangle that doesn't have any horizontal sides. **We can support students to understand that it doesn't matter which side of the triangle you choose for the base, but it does matter that you position the height at right angles, relative to that base.**

Example 3: Area of a parallelogram

Establishing formulae for calculating the area of a parallelogram

Ask students about Figure 7:

Which is bigger, the area of the parallelogram or the area of the rectangle?

If students recognise the areas of the rectangle/parallelogram pairs to be equal, then say:

Prove it to me/convince me!

If students don't recognise the areas of the rectangle/parallelogram pairs to be equal, then say:

I think they are the same as each other.

Students can prove you right or prove you wrong. Either way, they are proving their thinking.

Encourage students to find the simplest way to rearrange the parallelogram to form its partner rectangle. Figure 8 shows that it's possible to make a vertical cut and move that piece of the parallelogram to the opposite end of the shape to form a rectangle.

We can create opportunities for students to generalise, by asking:

Is there a rule that you could use to describe a way to work out the area of a parallelogram?

What if you change the angles within the parallelogram? Does your rule work for parallelograms with different internal angles?

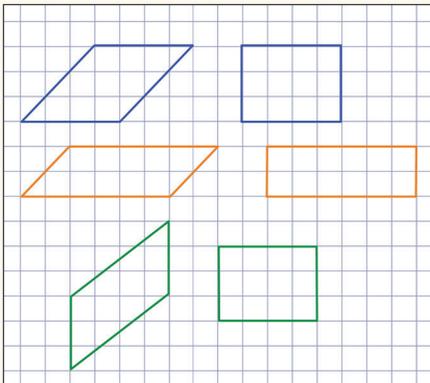


Figure 7

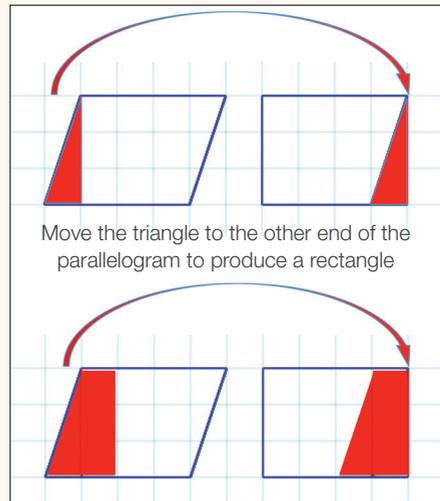


Figure 8

Example 4: Converting between units

Length, mass and capacity—connecting decimals to the metric system

Students will have been using metric units of measure since Year 3 (and possibly earlier), so when we begin to focus on converting between different units, we can use their prior learning. We can ask students to complete tables, such as those in Figure 9.

These tables can be completed from recall or students can work out solutions by looking at containers, scales and tape measures.

We can then ask questions such as:

What connections can you see between litres and millilitres?

What operation (+ – × ÷) could be used to change from the number of litres to the number of millilitres? The operation that you choose must work on every row of your table.

Is there a rule that you could use to change from the number of litres to the number of millilitres? How could you write that rule down (in words and perhaps in symbols for Year 7)?

What if you didn't have a whole number of litres, say $1\frac{1}{2}$ litres, does your rule still work? How do you know? Convince me!

What if you wanted to change from millilitres to litres, does your rule still work?

litres	millilitres
1	
2	
3	
4	

kilograms	grams
1	
2	
3	
4	

tonnes	kilograms
1	
2	
3	
4	

centimetres	millimetres
1	
2	
3	
4	

metres	centimetres
1	
2	
3	
4	

kilometres	metres
1	
2	
3	
4	

Figure 9

We can make connections to decimal representations:

What if you use numbers other than whole numbers and halves, does your rule still work?

Try changing $\frac{1}{4}$ (0.25) of a litre into millilitres.

Try changing 200 ml into litres etc.

It is important to point out to students that if they remember the basic connection:

1 L = 1000 ml, they will always be able to work out what to do (a rule) to change between litres and millilitres.

To do this, just think:

What do I need to \times or \div by, to get from 1 to 1000? I need to multiply by 1000 to get from 1 to 1000, so, to get from Litres to millilitres I must multiply by 1000.

What do I need to \times or \div by, to get from 1000 to 1? I need to divide by 1000 to get from 1000 to 1, so, to get from millilitres to Litres I must divide by 1000.

Example 5: Connecting volume and capacity

Connections between units of different attributes (volume and capacity)

The metric system is beautifully constructed, such that there is a relationship between the different units. For example, 1 cm^3 is the same as 1 ml and for water this quantity has a mass of 1 g ! At this stage of development it is only necessary for students to understand the connection between volume and capacity. For example: $1 \text{ ml} = 1 \text{ cm}^3$.

Submerging MAB blocks in a measuring cylinder (like those shown in Figure 10) containing water and observing the effect on the water level, is a very easy way to establish this connection, without instructing students. We can ask students to submerge different quantities of MAB blocks in a measuring cylinder and ask:

What connection do you notice between the number of centimetre cubes that you place in the measuring cylinder and the number of millilitres the water level rises by?



Figure 10

Take care to use MAB cubes/lengths that have been cut to metric measurements. Cubes should be $1 \text{ cm} \times 1 \text{ cm} \times 1 \text{ cm}$ and the 10 lengths should be $1 \text{ cm} \times 1 \text{ cm} \times 10 \text{ cm}$. It's worth checking your MAB blocks before using them in this way.



For larger volumes

Using a bucket of water, filled to the point of overflow, ask students to submerge a 1000 cm^3 (10 cm x 10 cm x 10 cm) MAB block in the water. Capture (in a tray) the water that flows over the edge of the bucket and measure this in a measuring jug.

Before doing this ask students:

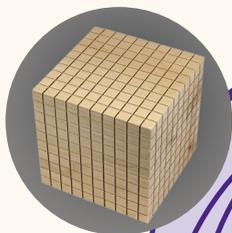
How many millilitres do you think will be displaced? Why?

Encourage students to make connections to familiar items such as cans and bottles of soft drink. Have a range of items for students to handle as they make their predications.

We are aiming for students to be familiar with the connection between volume and capacity:

$$1 \text{ cm}^3 = 1 \text{ ml, therefore } 1000 \text{ cm}^3 = 1000 \text{ ml} = 1 \text{ L}$$

Once students have established this connection, displacement becomes an accessible way to approximate the volume of any solid object. Given that at this stage of development students are only familiar with the formula for the volume of a rectangular prism, displacement facilitates students ordering a collection of objects by volume, even if they cannot calculate the volume.



WORKING IN AUTHENTIC CONTEXTS—CREATING CONNECTED LEARNING EXPERIENCES

When students carry out practical exercises such as this, they are unlikely to gather perfectly accurate data. There will be a range of solutions for the same displacement carried out by different students or by the same students on different occasions.

This is a perfect opportunity to work with averages in a meaningful context. (Year 7)



Example 6: Relationships between attributes

Area and perimeter

When students enlarge shapes (AC: Mathematics Year 5), say by a scale factor of 2, they double the length of the sides of the image and therefore:

- the perimeter doubles ($\times 2$)
- the area quadruples ($\times 4$)
- if you work with 3D objects the volume of the enlarged object is 8 times the volume of the original object ($\times 8$).

An understanding of the relationships described here, can be developed, rather than instructed.



The website *NRICH enriching mathematics* provides a detailed introduction to inquiry about these

relationships and as always, it also provides a sample of correct possible solutions that have been submitted by students from across the world. 'Growing rectangles' link: <http://nrich.maths.org/6923>

However, an introduction to this learning, could be as simple as showing students images such as those in Figure 11 and asking:

What relationships can you see between the small shape and the larger shape?

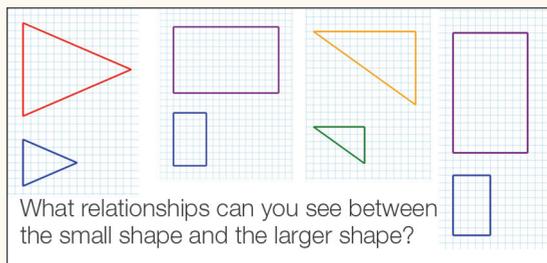


Figure 11

This question provides an opportunity for students to use language related to shape, angle, symmetry and fractions as well as area, perimeter and enlargement. We can allow time for discussion and value the connections that students identify across a range of topics.

Ask students to focus on the fact that in each of these pairs, the length of the sides has doubled. If students have not noticed or commented on the length of the sides, the perimeter or the area then ask:

If the lengths of the sides are twice as long, what does that mean for the perimeter?

If students don't know the word perimeter then trace the perimeter of the shapes with your finger and say that you are tracing the perimeter, then ask the student what they think the word perimeter means.

If students suggest that the perimeter has doubled, we can ask:

Are you sure? Do you want to check? Is it the same for all of the pairs?

We can then challenge thinking about area, by asking:

How many times larger do you think the area is (in the enlarged shape)? Are you sure? Do you want to check? Is it the same for all of the pairs? How can you prove that to me/to yourself?

Do you think these connections work for all rectangles/triangles/polygons, or have I chosen special cases to show you?

What other questions could you ask yourself?

Possible prompt:

What if you enlarge a shape by a different scale factor? What will change?

Challenge students to infer. Ask:

Now that you know the relationship (between scale factor and perimeter and between scale factor and area) for a scale factor of two, what do you think the relationship will be for a scale factor of three?

Challenge students to generalise. Ask:

Is there a rule that connects the scale factor of enlargement and the perimeter of the enlarged shape?

Is there a rule that connects the scale factor of enlargement and the area of the enlarged shape?

Can you express your rule in words and symbols and then using algebra.

For example, if the scale factor is 'y', and the original perimeter is 'p', the new perimeter would be 'p times y', which we could write as; $p \times y$ or even as py . If the scale factor is 'y' and the original area is 'n' the new area would be 'n times y times y'. We could write this as; $n \times y \times y$ or as; nyy or as ny^2 .

We can challenge students to reason, by asking:

Why does this relationship work in this way?

Extension for Year 6–7 students

These relationships could be graphed; for example, with 'scale factor of enlargement' on the x-axis and the 'multiplier for the perimeter' on the y-axis.

Connect to statistical representations

Consider how this multiplier effect is used in advertising to mislead the consumer!

Example 7: Rectangles—connecting area and perimeter

Applying formulae for calculating the area of a rectangle

A rectangle has an area of 24 units² (replace this with mm² or cm² or m² or leave your students to suggest what the units might be and take the opportunity to discuss what those different areas would look like). The sides of the rectangle are a whole number of units.

Ask students:

*What's its perimeter? **Are there other possibilities?***

*How many possible rectangles did you find? **Convince someone** that you have found all possible combinations.*

Suggested extensions

Extension 1: Connecting to factors

Ask students:

Will there be as many possibilities for all values? For example, if the area is 30 cm², will there be more/less/the same number of possible rectangles?

Can you see any relationships between the value and the amount of combinations?

What do you notice about the areas that have the most possibilities/the least possibilities?

We can connect this to a developing understanding of factors.

Extension 2: Connecting to multiplying and dividing fractions and decimals, with and without, digital technologies

Stay with the value of 24 cm^2 , but don't use whole centimetre squares. We can ask:

What if the rectangle is $1\frac{1}{2}$ squares wide. Are there more possibilities now?

What if the rectangle was $1\frac{1}{4}$ squares wide? $1\frac{1}{3}$ squares wide. Are there more possibilities now?

Students could work backwards/use inverse relationship. For example, the area divided by one side length will give the other side length; $24 \div 1\frac{1}{2} = 16$. At this stage such a calculation would be carried out using a calculator.

Alternatively students could use a trial and improvement strategy. For example, $1\frac{1}{2} \times \square = 24$.

The advantage of introducing these calculation challenges when working in the context of area is that students can use their skills in counting by unit fractions to check their result against a diagram representing the problem. This provides a concrete representation for content that can otherwise appear abstract to many students.

BACKWARDS QUESTIONS

Backwards questions are really just trickier application (choose and use, fluency) questions. They are questions that involve students choosing and using appropriate mathematics, but with a twist. An example of the twist may be that the student is given the solution and is required to work back to a possible starting point. It may be that there is a missing piece of information.



Example 8: Volume of a rectangular prism

Applying formulae for calculating the volume of a rectangular prism

Ask students to visualise a rectangular prism. Then ask:

If your rectangular prism had a volume of 24 cm^3 , what could your prism look like?

Are there other possibilities?

By Year 7, students should be able to solve a problem such as this by applying their understanding of the formula for calculating the volume of a rectangular prism, but equipment could be introduced as a tool for differentiation.

This problem could be extended in similar ways to Example 7.

Example 9: Area of triangles and parallelograms

Applying formulae for calculating the area of triangles and parallelograms

The area of a triangle is 24 cm^2 . The length of its base is 8 cm . Ask students:

What's the height of the triangle?

Draw a sketch of the triangle.

Is there another way that the triangle could be drawn?

Figure 12 shows a selection of triangles with the same base and the same area. Notice they all have the same height.

The same principle can be applied to a parallelogram question. The area of a parallelogram is 24 cm^2 . The length of its base is 8 cm . Ask students:

What's the height of the parallelogram?

Draw a sketch of the parallelogram.

Is there another way that the parallelogram could be drawn?

Figure 13 shows a selection of parallelograms with the same base and the same area. Notice they all have the same height.

If students struggle to see the different possible representations, prompt them with examples (such as those in Figures 12 and 13) and challenge them to create further examples.

Reasoning can be developed by asking:

How can you prove that all of these different representations have the same area?

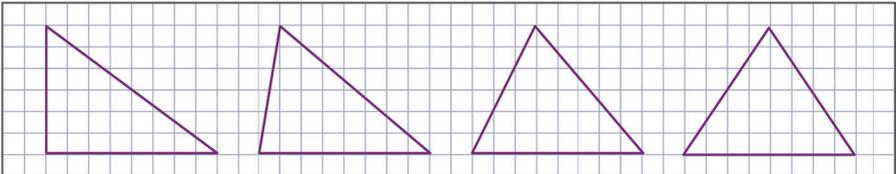


Figure 12

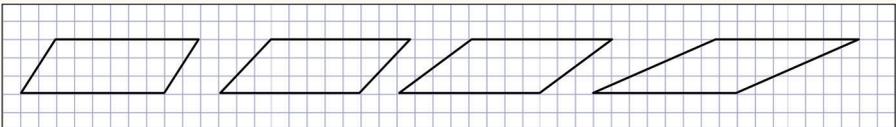


Figure 13

Example 10: Calculations using the four operations in the context of measurement

Ask questions that generate to the need to solve a problem in the form:

$$\text{value} \times \square = \text{value}$$

or

$$\text{value} \div \square = \text{value}$$

or

$$\text{value} + \square = \text{value}$$

or

$$\text{value} - \square = \text{value}$$

For example:

$$300 \text{ ml} \times \square = 1500 \text{ ml}$$

or

$$1500 \text{ g} \div \square = 500 \text{ g}$$

or

$$500 \text{ m} + \square = 1200 \text{ m}$$

or

$$230 \text{ cm} - \square = 65 \text{ cm}$$

$$300 \text{ ml} \times \square = 1.5 \text{ L}$$

or

$$1500 \text{ g} \div \square = \frac{1}{2} \text{ kg}$$

or

$$\frac{1}{2} \text{ km} + \square = 1200 \text{ m}$$

or

$$230 \text{ cm}^3 - \square = 0.65 \text{ m}$$

Notice that the challenge has been increased in the second column of questions, as students need to change between different units, as well as, manage the calculation.

Example 11: Ordering in different ways

Using different attributes

Ask students to order a given collection of items from smallest to largest.

Include items of different shapes, dimensions and densities. One suggested collection, shown in Figure 14, includes a hand, a can of soft drink, an iPhone, a tennis ball and a slice of bread.

To be able to order a collection such as this, students will first need to decide on an attribute to measure.

Once students have ordered the objects in one way, we can ask:

Is there another way to order this collection from smallest to largest?



Figure 14

Ordering this collection by volume will require different approaches for the different items.

Displacement could be used for some items, but not others. Some items could be approximated to a rectangular prism and the formula could be applied, some items could be modeled using MAB blocks. Some items lend themselves to more than one method, so this would provide a check.

Realistically this becomes a problem solving task for most students.



Proficiency: Problem Solving

Proficiency emphasis and what questions to ask to activate it in your students (Examples 12–14)

The AC: Mathematics defines the proficiency of ‘Problem Solving’ as:

What the
AC says



Proficiency: Problem Solving

Students develop the ability to make choices, interpret, formulate, model and investigate problem situations, and communicate solutions effectively. Students formulate and solve problems when they use mathematics to represent unfamiliar or meaningful situations, when they design investigations and plan their approaches, when they apply their existing strategies to seek solutions, and when they verify that their answers are reasonable.

BitL tool



There are four BitL questions associated with this proficiency. The four questions are:

How can you interpret?

In what ways can you model and plan?

In what ways can you solve and check?

Reflect?

Examples
overview



Interpret; Model and plan; Solve and check; Reflect

The Problem Solving section of the BitL tool works differently to the rest of the maths BitL tool—the sequence of four questions reflect the problem solving process. Therefore, unlike the other three proficiencies, we have not provided examples categorised under each question.

Example 12: 2-digit x 2-digit multiplication

Using student established formulae for calculating areas of rectangles

Example 13: Stacking cups

Problem solving using metric units of length

Example 14: A kilo of 10c coins

Problem solving with mass and volume



What's the difference between meaningful and unfamiliar problems?

Meaningful problems

Meaningful problems are those in which the mathematics and strategy being applied is familiar to the student. They have solved a problem similar to it before, so they can use the same strategy. The student understands the required content.

Unfamiliar problems

Unfamiliar problems include:

- Problems for which the students would not be able to say that they had done a similar example previously, they would therefore need to create an approach/develop a strategy.
- Problems in which the students develop a new piece of knowledge. They begin the problem by applying the knowledge/skills that they have and they complete the problem having re-combined that knowledge to form a new piece of understanding.

BUILDING RESILIENCE

Unfamiliar problems tend to provoke a response of, 'I don't know' or 'I'm not sure'. Students respond differently to this feeling. Some students shut down, others begin to ask, 'But how could I work that out?'

In developing powerful learners we are aiming for all of our students to learn that 'not knowing' is the *beginning of a learning opportunity* and that the first move they need to make on the journey to *finding out more* is to ask, 'What could I do to work this out?'



Example 12: 2-digit x 2-digit multiplication

Using student established formulae for calculating areas of rectangles

Provide students with a large rectangle, such as the one in Figure 15 and ask:

What's the area of this rectangle? You can't use a calculator!

Interpret

What have you been asked to calculate? What information is helpful/ no use? Are the squares useful or not? What would you do if it was a smaller rectangle?

Establish that the student understands that the area of a rectangle can be worked out by counting squares or by multiplying the length by the width.

Model and plan

Do you have an idea? How might you start? What equipment might be helpful? Would it help if you split the rectangle up into smaller rectangles?

Are there different ways that you could do that? What do you think would be easiest/most efficient/always work for you? Speak to someone who you think is being a good problem solver today and ask them to show you what they are trying.



Figure 15

We can become more supportive if students cannot make progress.

How could you partition the number 23? How could you partition the number 42? Do you have a new idea now? Draw lines on your rectangle to show those partitions.

What is 2×3 ? Where could you see a rectangle that is 2×3 on your grid?

Solve and check

How is multiplying by 20 similar to multiplying by 2? How could you investigate that? What about multiplying by 30...what's that similar to? What about multiplying by 40...what's that similar to?

Is there another way that you could have solved this problem? Could you split the rectangle up in a different way?

Reflect

What was your most efficient method? Did other people solve this problem in a different way? Is there something that you would do differently next time? Will your method work for any values? What if it wasn't drawn to scale, could you still use your method?

Figure 16 supports students to establish a grid method for multiplication. Relating this grid method to the context of area facilitates students building an understanding of why the grid method works.

Once students realise that drawing the multiplication problem to scale is not necessary, they can set out the calculation 42×23 , as shown in Figure 17. They could extend this process to 3-digit by 2-digit multiplication.

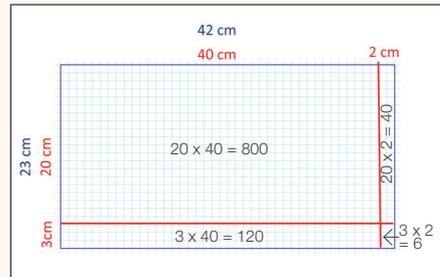


Figure 16

x	40	2	800
20	800	40	120 +
3	120	6	40
			6
			<u>966</u>

Figure 17

Example 13: Stacking cups

Problem solving using metric units of length

The images in Figure 18 are stills from a video by Andrew Stadel on ‘Dan Meyer’s 101 questions’ <http://www.101qs.com/1897-stacking-cups--act-1>



Figure 18

The growth in the overall height of the stacked cups could be graphed, with ‘number of cups’ on the x-axis and the ‘tower height’ on the y-axis. Notice, the variable we are controlling (ie the cups) goes on the x-axis.

Sharing a video or an image with students and asking what questions come to mind, can be an excellent way to engage students in problem solving. There are a wealth of examples that can be accessed free of charge on the Dan Meyer’s blog (<http://www.101qs.com>).

Example 14: A kilo of 10c coins

Problem solving with mass and volume

We can introduce a problem about mass and money in a number of ways. We could simply say:

I think that a kilo of coins will be worth about \$10. What do you think? What size container would you need to hold a kilo of coins?

Or

I think that a kilo of 10c coins would be worth about \$10. What do you think? What size container would you need to hold a kilo of 10c coins?

Alternatively, you could collect a large bag of (5c) coins and engage students by asking them to guess how many coins are in the bag. Allow students to weigh the bag of coins and challenge them to use that information to work out approximately how many coins are actually in the bag.

We can really make a point about learning to trust mathematics by not allowing students to count the coins to check their answer. Instead challenge them to share their thinking and calculations with each other until they are confident they are correct. That's the power of calculation. You don't need to count to check!



Proficiency: Reasoning

Proficiency emphasis and what questions to ask to activate it in your students (Examples 15–16)

The AC: Mathematics defines the proficiency of ‘Reasoning’ as:

What the
AC says



Proficiency: Reasoning

Students develop an increasingly sophisticated capacity for logical thought and actions, such as analysing, proving, evaluating, explaining, inferring, justifying and generalising. Students are reasoning mathematically when they explain their thinking, when they deduce and justify strategies used and conclusions reached, when they adapt the known to the unknown, when they transfer learning from one context to another, when they prove that something is true or false and when they compare and contrast related ideas and explain their choices.

BitL tool



There are four BitL questions associated with this proficiency. They reflect the student actions as described in the AC: Mathematics. The four questions are:

Q1 In what ways can you prove?

Q2 In what ways can you communicate?

Q3 In what ways can we generalise?

Q4 What can you infer?

Q1

Examples
overview



In what ways can you prove?

The intent of this question is to promote learning design in which students justify their (mathematical) actions in order to develop an increasingly sophisticated capacity for logical thought and action.

We can look for opportunities to ask students to prove their thinking to you, to themselves, or to their peers. Whether an answer is right or wrong, we can be ready to say:

Are you sure? Convince me, convince someone who thinks differently to you.

Convince me, often builds on from a sequence of thinking.

Examples 2, 3, 4, 7 and 9 all ask students to convince someone about their ideas.

Q2

Examples overview



In what ways can you communicate?

The intent of this question is to promote learning design that intentionally plans for students to compare, contrast and evaluate different ideas and approaches and to explain their reasoning using increasingly sophisticated mathematical terminology and conventions.

We must intentionally plan for communication to be in different modes, including written, verbal, symbolic and descriptive.

Example 15: Why is it not...?

Area of a triangle

Q3

Examples overview



In what ways can we generalise?

The intent of this question is to promote learning design that intentionally plans for students to look for generalisations. As students learn to generalise, they see mathematics as a set of connected ideas, rather than lots of separate rules and processes.

Establishing formulae requires generalisation. Telling students formulae removes this opportunity.

Examples 1, 2, 3, 4, 5 and 6 begin with challenging students to notice patterns and connections. The questioning in each of these examples is developed to the point where students are supported, and challenged, to make a generalisation.

Q4

Examples overview



What can you infer?

The intent of this question is to promote learning design in which students think about the implications of information they have, and develop flexibility to refine their thinking as more information is used.

Example 16: Gulliver's table

Selecting and estimating using appropriate units

Example 15: Why is it not...?

Area of a triangle

In addition to looking for opportunities to challenge students to become increasingly clear in their use of subject specific language, both written and spoken, we can also ask questions of the form:

Why is it not...?

Such questions provide an opportunity for us to hear our students' thinking.

For example, refer to Figure 19 and ask students:

Why is the area of this triangle not 68 cm^2 ?

This will allow you to see if students understand the application of the formula for calculating the area of a triangle. Asking if the actual area would be smaller or larger than 68 cm^2 would require the student to reason further.

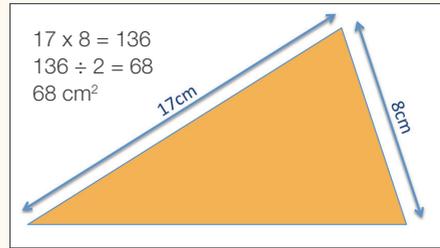


Figure 19

Extension activity

This question could then lead to a problem solving activity in which students keep two side-lengths of a triangle fixed, but change the angle between these two sides and observe when the area becomes larger/smaller, compared to when the two sides are at right angles to each other.

Example 16: Gulliver's table

Selecting and estimating using appropriate units

Photographs, such as this one in Figure 20 by Statler Hilton from 'Dan Meyer's 101 questions' (<http://www.101qs.com/434-pleasetake-a-seat-gulliver>) can be used to engage students in asking their own questions.

In response to this image, students might ask questions about the dimensions of the table and chair or about the comparison between this and their own tables and chairs. Students might ask questions such as:

Is that table/chair taller than our classroom?

Would that table/chair fit in to our gym?

How many people could sit on that chair?

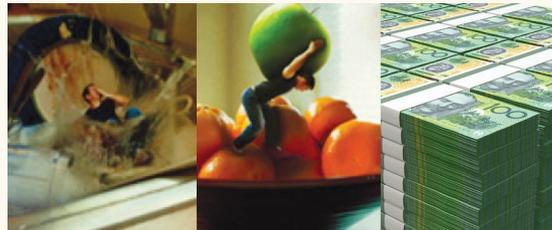
Students could reference the man standing by the right-hand leg of the table and make the assumption that he is an average height male. The height of an average male could be identified through research (appropriate for Year 5–6 students) or through data collection and calculating averages (appropriate for Year 6–7 students).

Students would also need to consider which adult/child to use as their measure. Could they use any? How will they make a measure for that adult/child (out of string/out of paper/find an object the same size etc).



Figure 20

Working with Year 5 to 7 students, using any of the images shown in the example below, is likely to create a problem solving challenge for students. What is special in these cases is that the challenge begins with the need to make inferences in order to gain data to work with, rather than simply identifying the data.



Proficiency: Fluency

Proficiency emphasis and what questions to ask to activate it in your students (Examples 17–24)

The AC: Mathematics defines the proficiency of ‘Fluency’ as:

What the
AC says



Proficiency: Fluency

Students develop skills in choosing appropriate procedures, carrying out procedures flexibly, accurately, efficiently and appropriately, and recalling factual knowledge and concepts readily. Students are fluent when they calculate answers efficiently, when they recognise robust ways of answering questions, when they choose appropriate methods and approximations, when they recall definitions and regularly use facts, and when they can manipulate expressions and equations to find solutions.

BitL tool



There are two BitL questions associated with this proficiency. They reflect the student actions as described in the AC: Mathematics. The two questions are:

Q1 What can you recall?

Q2 Can you choose and use flexibly?

Q1

Overview



What can you recall?

The intent of this question is to promote learning design in which students develop their capacity to recall mathematical information and processes.

Many worksheets are fluency driven exercises. Practice is a necessary part of developing fluency, but we should be purposeful about addressing the other proficiencies and be purposeful about developing fluency.

There are many worksheets that require recall of measurement related information. Instead of examples, in this section we have provided guidance about the language, facts and connections that we can aim for students to have recall of by the end of Year 7.

Q2

Examples overview



Can you choose and use flexibly?

The intent of this question is to promote learning design that intentionally plans for students to experience contexts in which they can choose to use knowledge and skills that they have developed.

Example 17: Making application more challenging

Four techniques

Example 18: Probably measurement

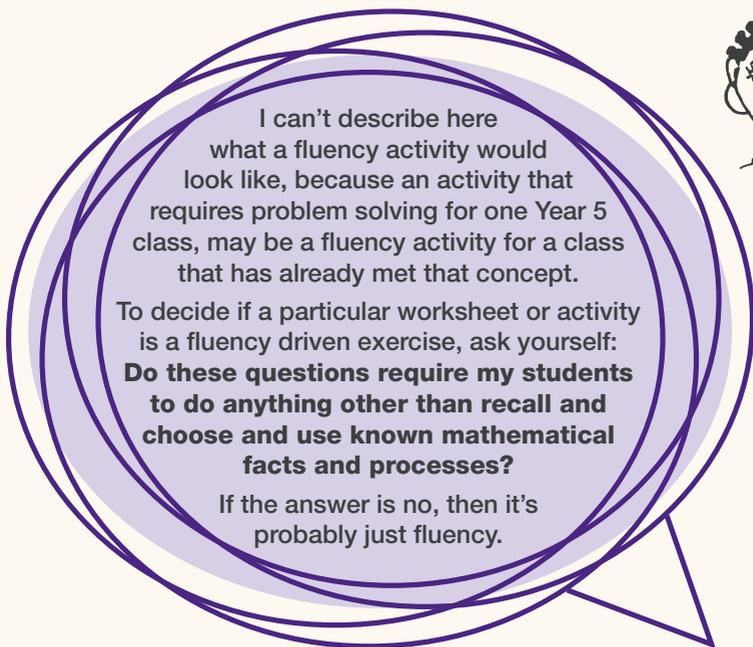
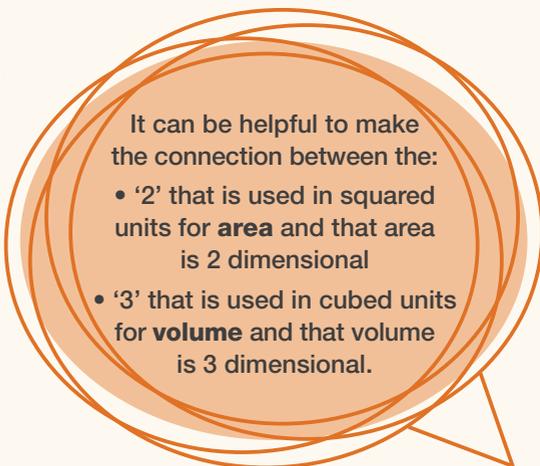
Bringing together units of measure and probability

What can you recall?

Abbreviations

Challenge and support students to recall abbreviations for each of the metric units they use, for example:

- **Length:** mm, cm, m, km
- **Area:** mm², cm², m², km²
- **Volume:** mm³, cm³, m³, km³
- **Capacity:** ml, L
- **Mass:** mg, g, kg, t (mt or T).



Prefixes

Learning and understanding these three prefixes, prevents students from memorising a much greater number of equivalent measures.

1 Knowing that **milli means thousandth** helps with understanding:

- millilitre means thousandth of a litre
- millimetre means thousandth of a metre
- milligram means thousandth of a gram.

2 Knowing that **centi means hundredth** helps with understanding:

- centimetre means hundredth of a metre
- centilitre means hundredth of a litre.

3 Knowing that **kilo means thousand** helps with understanding:

- kilometre means thousand metres
- kilogram means thousand grams
- kilolitre means thousand litres.

These prefixes can be ‘played with’ by asking quick questions such as:

What could you call a hundredth of a Helen (or any child’s name)? A Centi-Helen.

What could you call a thousand Helens? A Kilo-Helen.

What’s a milli-Helen worth? A thousandth of a Helen.

Challenge and support students to recall equivalent measures. To be able to change between different units of metric measures, students will need to recall that:

- 1 cm = 10 mm
- 1 m = 100 cm
- 1 km = 1000 m
- 1 L = 1000 ml
- 1 g = 1000 mg
- 1 kg = 1000 g
- 1 t = 1000 kg.



If understanding and recall about prefixes is developed, when students come across a less frequently used measure such as centilitres, they will be able to interpret the meaning as ‘hundredths of a litre’.



Watch out for students who think you are saying ‘killer’ instead of ‘kilo’. Maths concerns enough children without them thinking there are killer metres out there!

Frequent opportunities to speak, hear, read and record language, symbols and facts related to units of measure will help students to commit this information to memory.

Developing the capacity to recall information such as this can be done through:

- the use of word walls
- insisting on the use of units in written work
- quick questions responded to on white boards rather than in quizzes on paper.



Estimation skills can be supported by recall

As estimation can be made by comparison to a known unit, it's useful for students to be able to recall a range of measurements that connect to their size, or to objects that are familiar to them. For example:

Estimating length



I remember that my little finger is about 1 cm wide. That helps me to estimate small lengths.



I want to find out how wide my hand is, so I can use it to help me to estimate length. I also think it would be good to know what it feels like to step out $\frac{1}{2}$ a metre or a whole metre, then I could estimate larger distances.



When I am estimating length, I find it useful to know my own height and the height of the door.

Estimating capacity



Lots of my students can recall the capacity of soft drink containers, but I've noticed they don't automatically use that knowledge. Perhaps it's because they learnt it outside of mathematics lessons, that they are unaware of the value when solving mathematics problems.

Estimating mass



I let my students weigh and label items they use frequently. I think this helps their recall of the mass of a few familiar objects and they can use this knowledge when they are estimating. Also, when I'm cooking with students, I support them to notice measurements of mass, both as they weigh produce and as they read labels on pre-packed items such as butter.

Example 17: Making application more challenging



Four techniques

Things to consider when writing or selecting questions in which students will choose and use their understanding of units of measure:

- 1 For perimeter, area and volume problems, present shapes in different orientations. This is described in the 'shape narrative', for example, sometimes draw rectangles with sides that are not horizontal and vertical or draw triangles where no side is horizontal.
- 2 Give additional, unnecessary information, rather than just the information that is required. This shakes fragile understanding and reveals underdeveloped conceptual understanding (Figure 21).

Notice the use of labelling conventions in this question. Capital letters are always used to label vertices.

- 3 Sometimes ask questions that require students to take measurements themselves (Figure 22).
- 4 Give enough information for the necessary measurements to be calculated, but don't give all of the information directly (Figure 23).

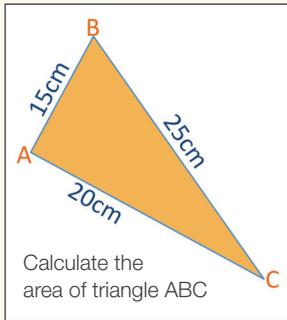


Figure 21

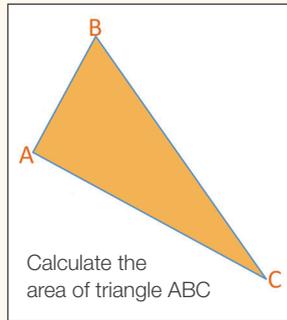


Figure 22

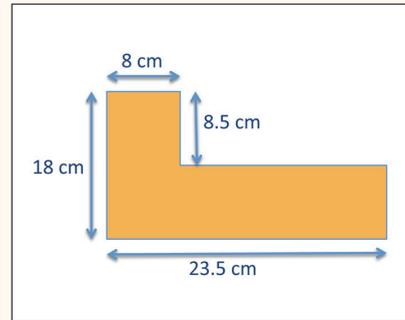


Figure 23

Example 18: Probability measurement

Bringing together units of measure and probability

Ask students to pose and investigate probability questions that relate to units of measurement. For example:

- If we randomly select a student from our Year 1 class, what's the probability that the child will be between 1.2 and 1.3 metres tall?
- If we randomly select a student from our Year 7 class, what's the probability that they will be able to inflate a balloon to a volume greater than 600 cm^3 in one breath?

The probability component of these examples may involve problem solving for students, if it is new learning for them. However the measurement component is straightforward application of using units of measurement.



Connections between ‘using units of measurement’ and other maths content

There are many opportunities to connect to other content in the AC: Mathematics, when we use measurement as a starting point.

Here are just some of the possible connections that can be made:

Mathematics: Year 5	
Whilst working with units of measurement, connections can be made to:	How the connection might be made:
Recognise, model, represent and order numbers to at least 10 000.	Working with area provides an opportunity for students to identify the area/grid method for multiplying large numbers. Refer to Example 12 .
Connect three-dimensional objects with their nets and other two-dimensional representations.	Create nets of rectangular prisms with the same volume as each other and ask if their nets will all have the same surface area as each other.
Apply the enlargement transformation to familiar two dimensional shapes and explore the properties of the resulting image compared with the original.	Refer to Example 6 .
Pose questions and collect categorical or numerical data by observation or survey.	Refer to Example 5 and Example 18 .
Construct displays, including column graphs, dot plots and tables, appropriate for data type, with and without the use of digital technologies.	Use examples that relate to units of measurement. Refer to Example 6 , Example 13 and Example 18 .
Describe and interpret different data sets in context.	Use examples that relate to units of measurement.

Mathematics: Year 6

Whilst working with units of measurement, connections can be made to:	How the connection might be made:
Find a simple fraction of a quantity where the result is a whole number, with and without, digital technologies.	Calculate fractions of lengths, areas, volumes etc.
Continue and create sequences involving whole numbers, fractions and decimals. Describe the rule used to create the sequence.	Refer to Example 13 .
Construct simple prisms and pyramids.	With a specific surface-area or volume (for rectangular prisms). Extend Example 8 to include this content description.
Describe probabilities using fractions, decimals and percentages.	Ask students to pose and investigate probability questions that relate to units of measurement. Refer to Example 18 .
Interpret and compare a range of data displays.	Use examples that relate to units of measurement.
Interpret secondary data presented in digital media and elsewhere.	Use examples that relate to units of measurement.

Mathematics: Year 7

Whilst working with units of measurement, connections can be made to:	How the connection might be made:
Multiply and divide fractions and decimals using efficient written strategies and digital technologies.	Refer to Example 7: Extension 2 and Example 8 (if extended) .
Express one quantity as a fraction of another, with and without, the use of digital technologies.	Example 16 could be used to achieve this.
Introduce the concept of variables as a way of representing numbers using letters.	Refer to Example 1 , Example 2 , Example 3 and Example 6 .
Draw different views of prisms and solids formed from combinations of prisms.	Consider the surface area of a face on the actual object, compared to the apparent area of the same face on the plan/front or side view.
Assign probabilities to the outcomes of events and determine probabilities for events.	Ask students to pose and investigate probability questions that relate to units of measurement. Refer to Example 18 .
Calculate mean, median, mode and range for sets of data. Interpret these statistics in the context of data.	Refer to Example 5 , Example 14 (if students weigh the coins to establish the mass, rather than google the mass of a 10c coin), Example 16 (if students collect measurements to infer a height for a person in the image).
Identify and investigate issues involving numerical data collected from primary and secondary sources.	Use examples that relate to units of measurement.
Construct and compare a range of data displays including stem-and-leaf plots and dot plots.	Use examples that relate to units of measurement.
Describe and interpret data displays using median, mean and range.	Use examples that relate to units of measurement.

‘Using units of measurement’ from Foundation to Year 10A

The AC: Mathematics year level content descriptions shown here have been colour coded to highlight the following curriculum aspects of working with measurement:

Using informal units for direct or indirect comparisons

From Foundation to Year 2 students focus on informal units of measurement.

Using standard metric units

From Year 3 to Year 8 students develop their understanding of metric units of measure. This begins with the use of familiar metric units and extends to include a greater range of metric units and the flexibility to convert between different units.

Establishing and applying formulae

From Year 5 to Year 10 students establish and use formulae of increasing complexity relating to perimeter, area and volume.

Estimating

Australian Curriculum references to estimation in relation to measurement lie entirely in the Numeracy Continuum.

Year level	‘Using units of measurement’ content description from the AC: Mathematics
Foundation	Use direct and indirect comparisons to decide which is longer, heavier or holds more, and explain reasoning in everyday language.
Year 1	Measure and compare the lengths and capacities of pairs of objects using uniform informal units.
Year 2	Compare and order several shapes and objects based on length, area, volume and capacity using appropriate uniform informal units.
Year 2	Compare masses of objects using balance scales.
Year 3	Measure, order and compare objects using familiar metric units of length, mass and capacity.
Year 3	Use scaled instruments to measure and compare lengths, masses, capacities and temperatures.
Year 4	Compare objects using familiar metric units of area and volume.
Year 5	Choose appropriate units of measurement for length, area, volume, capacity and mass.
Year 5	Calculate the perimeter and area of rectangles using familiar metric units.

Year 6	Connect decimal representations to the metric system.
Year 6	Convert between common metric units of length, mass and capacity.
Year 6	Solve problems involving the comparison of lengths and areas using appropriate units.
Year 6	Connect volume and capacity and their units of measurement.
Year 7	Establish the formulas for areas of rectangles, triangles and parallelograms and use these in problem solving.
Year 7	Calculate volumes of rectangular prisms.
Year 8	Choose appropriate units of measurement for area and volume and convert from one unit to another.
Year 8	Find perimeters and areas of parallelograms, trapeziums, rhombuses and kites.
Year 8	Investigate the relationship between features of circles such as circumference, area, radius and diameter. Use formulas to solve problems involving circumference and area.
Year 8	Develop the formulas for volumes of rectangular and triangular prisms and prisms in general. Use formulas to solve problems involving volume.
Year 9	Calculate the areas of composite shapes.
Year 9	Calculate the surface area and volume of cylinders and solve related problems.
Year 9	Solve problems involving the surface area and volume of right prisms.
Year 10	Solve problems involving surface area and volume for a range of prisms, cylinders and composite solids.
Year 10A	Solve problems involving surface area and volume of right pyramids, right cones, spheres and related composite solids.

Numeracy Continuum: Using measurement

End Year 2	Estimate, measure and order using direct and indirect comparisons and informal units to collect and record information about shapes and objects.
End Year 4	Estimate and measure with metric units: estimate, measure and compare the length, temperature, volume, capacity and mass of everyday objects using metric units and scaled instruments.
End Year 6	Estimate and measure with metric units: choose and use appropriate metric units for length, area, volume, capacity and mass to solve everyday problems.
End Year 8	Estimate and measure with metric units: convert between common metric units for volume and capacity and use perimeter, area and volume formulas to solve authentic problems.
End Year 10	Estimate and measure with metric units: solve complex problems involving surface area and volume of prisms and cylinders and composite solids.

Source: ACARA, Australian Curriculum: Mathematics, Version 8.0

Do you want to feel more confident about the maths you are teaching?

Do you want activities that support you to embed the proficiencies?

Do you want your students thinking mathematically rather than just doing maths?

If you answered **yes** to any of these questions, then this resource is for you.

Packed full of examples, along with questions you can ask students as they engage in their learning, this resource supports you to develop confidence in teaching the Australian Curriculum: Mathematics.

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