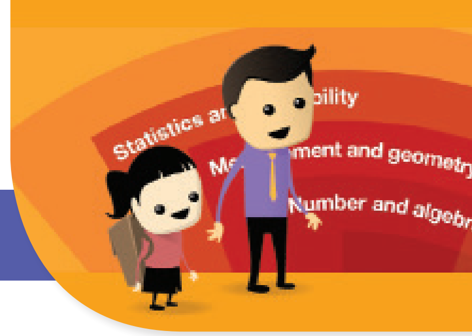

The BitL tool – mathematics years F–10

Fluency: Years F–2

What can you recall?

This is about remembering facts;

being able to name and identify numerals, simple shapes, symbols (such as +, -, =) recognising counting sequences, recognising Australian coins by value, being able to count forwards and backwards to 20/100/1000 (F/1/2) starting from any point; and being able to recall language related to time/ duration (eg days of the week, months of the year, seasons).

Pedagogical questions:

- How could you record that mathematically?
- Can you remember the name of that… (shape/number)?
- What words could you use to describe… (the time, the month, the date, the amount of time)?
- What is the value of… (a calculation that you would expect automatic recall of, eg number pairs to 10, to 100 etc)?
- How many…?

Examples

You have told me that there are twelve balls there. How could you record that?

Questions like this are intended to give students the opportunity to practise their fluency in recording numbers.

What is…? (single digit additions at appropriate level):
2 + 2   1 + 3   6 + 4 etc

Fluency: Years 3–4

What can you recall?

This is about remembering/identifying mathematical, names, shapes, symbols, facts and processes that are important to know when working with mathematical ideas.

Pedagogical questions:

- How could you record that mathematically?
- How could you... (eg calculate that)?
- How could you use a calculator to…?
- Can you remember a way to…?
- What is the value of… (a calculation that you would expect automatic recall of eg number pairs to 10, to 100, some times tables)?
- What is the name of…?
- What is the symbol for…?
- How many…?

Examples

What metric units of measurement are commonly used for length, area, volume, capacity and mass?

Recall multiplication and division facts up to and including 10 x 10.

Recall facts and definitions up to and including those used in the appropriate year level.

Can you remember a way to multiply a two digit number by a single digit number eg 36 x 7 ?

Can you remember a way to divide a two or three digit number by a single digit number (no remainders) eg 287 ÷ 7?

Notice that multiplication and division questions are also in the problem solving section. By the end of year 4 it would be appropriate for students to be able to respond to questions such as this as a fluency question. By this it is meant that students would be aware of several processes that they could apply. Prior to this it is crucial that students experience questions like this in a problem solving context, where they design possible processes and in doing so, make meaning of those processes.

Fluency: Years 5–6

What can you recall?

This is about remembering/identifying mathematical, names, shapes, symbols, facts, processes and formulas that are important to know when working with mathematical ideas.

Pedagogical questions:

- How could you record that mathematically?
- How could you... (eg calculate that)?
- How could you use a calculator to…?
- Can you remember a way to…?
- What is the value of… (a calculation that you would expect automatic recall of eg number pairs to 10, to 100, times tables)?
- What is the name of…?
- What is the symbol for…?
- What is the formula for…?
- How many…?
- How much…?

Examples

What metric units of measurement are used for length, area, volume, capacity and mass (including large and small increments)?

How many millimetres are there in a centimetre?
How many centimetres are there in a metre?
How many metres are there in a kilometre?

What is the name of... (show a range of 2D shapes and 3D objects, including prisms and pyramids)?

Can you remember a way to multiply two 2 digit numbers together eg 36 x 54?

Can you remember a way to divide a large number by a single digit number (with remainders)eg 2847 ÷ 7?

Fluency: Years 7–8

What can you recall?

This is about remembering/identifying mathematical, names, shapes, symbols, facts, processes and formulas that are important
to know when working with mathematical ideas.

Pedagogical questions:

- How could you record that mathematically?
- How could you... (eg calculate that)?
- How could you use a calculator to…?
- Can you remember a way to…?
- What is the value of… (a calculation that you would expect automatic recall of, eg times tables, some square numbers, square roots of perfect squares, some powers of 10 (eg 102=100, 103=1000)?
- What is the name of…?
- What is the symbol for…?
- What is the formula for…?
- How many…?
- How much…?

Examples

What formula can be used to calculate the area of a rectangle/a triangle?

Name the parts of a circle.

What is the value of 1/3 + 3/4?
What is the value of 3/4 - 1/3?
What is the value of 1/3 x 3/4?
What is the value of 3/4 ÷ 1/3?

Fluency: Years 9–10

What can you recall?

This is about remembering/identifying mathematical, names, shapes, symbols, facts, processes and formulas that are important
to know when working with mathematical ideas.

Pedagogical questions:

- How could you record that mathematically?
- How could you... (eg calculate that)?
- How could you use a calculator to…?
- Can you remember a way to…?
- What is the value of… (a calculation that you would expect automatic recall of eg times tables, some square numbers, square roots of perfect squares and some powers of 10)?
- What is the name of…?
- What is the symbol for…?
- What is the formula for…?
- How many…?
- How much…?

Examples

What formula can be used to calculate the volume of a rectangular prism/a triangular prism?

What is Pythagoras Theorem?

The BitL tool – mathematics years F–10

Fluency: Years F–2	Fluency: Years 3–4	Fluency: Years 5–6	Fluency: Years 7–8	Fluency: Years 9–10
Can you choose and use your maths flexibly? This is about choosing and using an appropriate action or appropriate mathematical information and language.	**Can you choose and use your maths flexibly?** To be able to choose and use mathematics efficiently students need to be able to recall processes and facts. Choosing and using is about selecting (age appropriate) processes, facts and mathematical language appropriate to the context.	**Can you choose and use your maths flexibly?** To be able to choose and use mathematics efficiently students need to be able to recall processes and facts. Choosing and using is about selecting (age appropriate) processes, facts and mathematical language appropriate to the context.	**Can you choose and use your maths flexibly?** To be able to choose and use mathematics efficiently students need to be able to recall processes and facts. Choosing and using is about selecting (age appropriate) processes, facts and mathematical language appropriate to the context.	**Can you choose and use your maths flexibly?** To be able to choose and use mathematics efficiently, students need to be able to recall processes and facts. Choosing and using is about selecting (age appropriate) processes, facts and mathematical language appropriate to the context.
Pedagogical questions: • Choose a way to record that mathematically. • Choose a way to (eg count/estimate/rename/measure/compare/order that). • What mathematical words can you use to describe…? • What would be an efficient way to...(count/add on/calculate/draw/record) that?	**Pedagogical questions:** • Choose a way to record that mathematically. • Choose a way to... (count/estimate/rename/measure/compare/order/calculate/partition/rearrange/regroup/record/show/represent that). • Use mathematical language to describe... • What would be an efficient way to... (count/measure/order/compare/add on/subtract/multiply/divide/calculate/draw/record)? • How could you... (partition/rearrange/regroup)? • How could you use a calculator to check your answer?	**Pedagogical questions:** • Choose a way to record that mathematically. • Choose a way to... (estimate/rename/measure/compare/order/calculate/partition/rearrange/regroup/record/show/represent) that. • Use mathematical language to describe… • What would be an efficient way to...(count/measure/order/compare/add on/subtract/multiply/divide/calculate/draw/record/simplify) that? • How could you use a calculator/computer to…?	**Pedagogical questions:** • Choose a way to record that mathematically. • Choose a way to (estimate/rename/measure/compare/order/calculate/partition/rearrange/regroup/record/show/represent that). • Use mathematical language to describe… • What would be an efficient way to... (count/measure/order/compare/add on/subtract/multiply/divide/calculate/draw/record/simplify/expand/rearrange/substitute that)? • How could you use a calculator/computer to…?	**Pedagogical questions:** • Choose a way to record that mathematically. • Choose a way to... (estimate/rename/measure/compare/order/calculate/record/show/represent that). • Use mathematical language to describe… • What would be an efficient way to... (count/measure/order/compare/add on/subtract/multiply/divide/calculate/draw/record/simplify/expand/rearrange/substitute that)? • How could you use a calculator/computer to…?
Examples Choose a way to arrange your counters, so that someone else can look at them and count them quickly/efficiently, or just see how many? *Notice that this fluency could have been developed as a result of students experiencing problem solving questions such as: Are there ways of arranging collections of counters, that make it easier to see at a glance how many counters there are?* Choose a way to find out what day of the week it will be on the first of April this year.	**Examples** Use an array to check your answer to the question 24 ÷ 3. There are 24 children in Pam's class. Each child is allowed to bring up to 4 guests to their open day. What is the maximum number of guests that will be at the open day? How could you use a calculator to work out the total number of books in three boxes with 36 books in each?	**Examples** Order a selection of angles from smallest to largest. Measure and record the size of each angle to verify the order that you have placed them in. Calculate how many millimetres there are in a kilometre. Calculate how many grams there are in a tonne.	**Examples** Calculate how many centimetre squares there are in a metre square. Calculate how many metres square there are in a kilometre square. Kym was driving to a concert. She used ¼ of a tank of fuel to get there, but she took a different route home and used 1/3 of a tank of fuel. If the tank was full before she left home and she didn't put any additional fuel in, what fraction of a tank does Kym have left? Factorise: $6xy + 2x$	**Examples** Calculate the surface area of a rectangular prism with dimensions 12cm, 15cm and 20cm. Factorise: $4x^2 + 6x + 2$ Calculate the length of the hypotenuse of a right angled triangle with opposite side 10cm and adjacent side 15cm. Write 3 rational numbers with values between 1 and 2. Evaluate $6^{1/2} \times (\sqrt[3]{64})^4$

The BitL tool – mathematics years F–10

Problem solving: Years F–2

Students benefit from working in a problem solving context in many aspects of the curriculum.

How can you interpret?

This is about creating meaning from the problem that has been presented. It is useful to have the students describe (in their own words) what they have been asked to do. Descriptions of the task could be oral or written, dependent on the students and the task.

Students should be encouraged to pose basic problems about their (immediate) world.

Pedagogical questions:

- What are you being asked to find out or show?
- What information is helpful?
- What information is not useful?

Closed questions can be useful to check if the student has accessed the information given in the question, for example

How many…?
How much…?
When…?

(These questions will vary depending on the context of the problem)

In what ways can you model and plan?

This is about describing a problem using mathematical concepts or language, then deciding what to do with that information.

Pedagogical questions:

- Do you have an idea?
- What could you try?
- Have you done a problem like this one before?
- How could you test your idea?
- How might you start?
- Can you represent the problem as a picture or by using equipment?
- Would (counting, a sum, a picture) help?
- Can you act it out?
- Can you represent the information using numbers and symbols?
- What questions could you ask (to find that out)?
- When we are being good problem solvers, what do we do to get started?
- Speak to someone who you think is being a good problem solver today. Ask them to show you what they are trying.

Problem solving: Years 3–4

Students benefit from working in a problem solving context in many aspects of the curriculum.

How can you interpret?

This is about creating meaning from the problem that has been presented or created by the student in response to curiosity about their world.

It is useful to have the students describe (in their own words) what they have been asked to do. Descriptions of the task could be oral or written, as appropriate for the students and the task.

Students should be encouraged to pose basic problems about their (immediate) world.

Pedagogical questions:

- What are you being asked to find out or show?
- What information is helpful?
- What information is not useful?

Closed questions can be useful to check if the student has accessed the information given in the question, for example
- How many…?
- How much…?
- When…?

(These questions will vary depending on the context of the problem)

In what ways can you model and plan?

This is about describing a problem mathematically. Across years 3 to 6 ideas are represented using models, pictures and symbols. The complexity of the pictures will develop from those representing an image of the problem (in years 3 and 4) to those that support thinking about the problem and are more abstract in appearance (in years 5 and 6).

It is important for students to think about how they will attempt to solve the problem, rather than rushing into taking measurements or making calculations without thinking first about how helpful that will be.

Pedagogical questions:

- Do you have an idea?
- What could you try?
- Have you done a problem like this one before?
- How could you test your idea?
- How might you start?
- Can you represent the problem as a picture or by using equipment?
- Would... (counting, a sum, a picture) help?
- Can you act it out?
- Can you represent the information using numbers and symbols?
- What questions could you ask?
- When we are being good problem solvers, what do we do to get started?
- Speak to someone who you think is being a good problem solver today. Ask them to show you what they are trying.

Problem solving: Years 5–6

Students benefit from working in a problem solving context in many aspects of the curriculum.

How can you interpret?

This is about creating meaning from the problem that has been presented or created by the student in response to curiosity about their world.

It is useful to have the students describe (in their own words) what they have been asked to do. Descriptions of the task could be oral or written, as appropriate for the students and the task.

Pedagogical questions:

- What are you being asked to find out, demonstrate or prove?
- What information is helpful?
- What information is not useful?
- What additional information would be useful?

Closed questions can be useful to check if the student has accessed the information given in the question, for example
- How many…?
- How much…?
- When…?

(These questions will vary depending on the context of the problem)

In what ways can you model and plan?

This is about describing a problem mathematically. Across years 3 to 6, ideas are represented using models, pictures and symbols. The complexity of the pictures will develop from those representing an image of the problem (in years 3 and 4) to those that support thinking about the problem and are more abstract in appearance (in years 5 and 6).

It is important to think about how you will attempt to solve the problem, rather than rushing into taking measurements or making calculations without first thinking about how helpful that will be.

Pedagogical questions:

- Do you have an idea?
- What could you try?
- Have you done a problem like this one before?
- How could you test your idea?
- How might you start?
- Can you represent the problem as a picture or by using equipment?
- Can you act it out?
- Can you represent the information using numbers and symbols?
- What questions could you ask (to find that out)?
- What information could you put in a diagram to support your thinking?
- When we are being good problem solvers, what do we do to get started?
- Speak to a peer. Ask them to show you what they are trying.

Problem solving: Years 7–8

Students benefit from working in a problem solving context in many aspects of the curriculum.

How can you interpret?

This is about creating meaning from the problem that has been presented or created by the student in response to curiosity about real world applications of mathematics that are relevant to the student.

It is useful to have the students describe (in their own words) what they have been asked to do. Descriptions of the task could be oral or written, as appropriate for the students and the task.

Pedagogical questions:

- What are you being asked to find out, demonstrate or prove?
- What information is helpful?
- What information is not useful?
- What additional information would be useful?

Closed questions can be useful to check if the student has accessed the information given in the question, for example
- How many…?
- How much…?
- When…?

(These questions will vary depending on the context of the problem)

In what ways can you model and plan?

This is about describing a problem mathematically. Across years 7 to 10, ideas are represented using models, diagrams and symbols. There is an increasing emphasis on abstract symbolic representation.

It is important for students to think about how they will attempt to solve the problem, rather than rushing into taking measurements or making calculations without first thinking about how helpful that will be.

Pedagogical questions:

- Do you have an idea?
- What could you try?
- Have you done a problem like this one before?
- How could you test your idea?
- How might you start?
- Can you represent the problem as a picture or by using equipment?
- Can you represent the information numerically or symbolically?
- What questions could you ask (to find that out)?
- What information could you put in a diagram to support your thinking?
- What strategies have you used in the past when you have been stuck?
- Speak to a peer. Ask them to show you what they are trying.

Problem solving: Years 9–10

Students benefit from working in a problem solving context in many aspects of the curriculum.

How can you interpret?

This is about creating meaning from the problem that has been presented or created by the student in response to curiosity about real world applications of mathematics that are relevant to the student.

It is useful to have the students describe (in their own words) what they have been asked to do. Descriptions of the task could be oral or written, as appropriate for the students and the task.

Pedagogical questions:

- What are you being asked to find out, demonstrate or prove?
- What information is helpful?
- What information is not useful?
- What additional information would be useful?

Closed questions can be useful to check if the student has accessed the information given in the question, for example
- How many…?
- How much…?
- When…?

(These questions will vary depending on the context of the problem)

In what ways can you model and plan?

This is about describing a problem mathematically. Across years 7 to 10 ideas are represented using models, diagrams and symbols. There is an increasing emphasis on abstract symbolic representation.

It is important to think about how you will attempt to solve the problem, rather than rushing into taking measurements or making calculations without first thinking about how helpful that will be.

Pedagogical questions:

- Do you have an idea?
- What could you try?
- Have you done a problem like this one before?
- How could you test your idea?
- How might you start?
- Can you represent the problem as a picture or by using equipment?
- Can you represent the information numerically or symbolically?
- What questions could you ask (to find that out)?
- What information could you put in a diagram to support your thinking?
- What strategies have you used in the past when you have been stuck?
- Speak to a peer. Ask them to show you what they are trying.

The BitL tool – mathematics years F–10

Problem solving: Years F–2 Students benefit from working in a problem solving context in many aspects of the curriculum.	**Problem solving: Years 3–4** Students benefit from working in a problem solving context in many aspects of the curriculum.	**Problem solving: Years 5–6** Students benefit from working in a problem solving context in many aspects of the curriculum.	**Problem solving: Years 7–8** Students benefit from working in a problem solving context in many aspects of the curriculum.	**Problem solving: Years 9–10** Students benefit from working in a problem solving context in many aspects of the curriculum.
In what ways can you solve and check? This is the mechanics of problem solving; the doing of calculations (the counting/adding/subtracting/sharing/grouping/building), and checking how appropriate the answer.	**In what ways can you solve and check?** This is the mechanics of problem solving; the doing of calculations (the counting/adding/subtracting/sharing/grouping/building) and checking how appropriate the answer is.	**In what ways can you solve and check?** This is the mechanics of problem solving - the doing of calculations (the adding/subtracting/sharing/grouping/constructing) and checking how appropriate the answer is.	**In what ways can you solve and check?** This is the mechanics of problem solving-the doing of calculations and checking how appropriate the answer.	**In what ways can you solve and check?** This is the mechanics of problem solving-the doing of calculations and checking how appropriate the answer is.
Pedagogical questions: • How can you... (count that/add those numbers together/subtract that amount)? • Does that seem right to you? • How can you check your answer? • Do other people think that too?	**Pedagogical questions:** • How can you... (add those numbers together/subtract that amount/multiply those amounts/divide those amount)? • What processes could you try? • Does that seem right to you? • How can you check your answer? • Do other people think that too?	**Pedagogical questions:** • How can you... (add those numbers together/subtract that amount/multiply those amounts/divide those amounts)? • What processes could you try? • Does that seem right to you? • How can you check your answer? • Do other people think that too?	**Pedagogical questions:** • How can you calculate that? • What processes could you try? • Does that seem right to you? • How can you check your answer? • Do other people think that too?	**Pedagogical questions:** • How can you calculate that? • What processes could you try? • Does that seem right to you? • How can you check your answer? • Do other people think that too?
Reflect Students need to reflect on how reasonable their answer is and also on the method that was used. There are different ways to solve problems and different ways to explain your thinking. At every stage of development, students benefit from sharing and reflecting on the strategies and reasoning of others.	**Reflect** Students need to reflect on how reasonable their solution is - they should consider if they have made an appropriate interpretation in relation to the context of the problem. There are different ways to solve problems and different ways to explain your thinking. At every stage of development, students benefit from sharing and reflecting on the strategies and reasoning of others.	**Reflect** Students need to reflect on how reasonable their solution is. They should consider if they have made an appropriate interpretation in relation to the context of the problem. There are different ways to solve problems and different ways to explain your thinking. At every stage of development, students benefit from sharing and reflecting on the strategies and reasoning of others.	**Reflect** Students need to reflect on how reasonable their solution is. They should consider if they have made an appropriate interpretation in relation to the context of the problem. There are different ways to solve problems and different ways to explain your thinking. At every stage of development, students benefit from sharing and reflecting on the strategies and reasoning of others.	**Reflect** Students need to reflect on how reasonable their solution is. They should consider if they have made an appropriate interpretation in relation to the context of the problem. There are different ways to solve problems and different ways to explain your thinking. At every stage of development, students benefit from sharing and reflecting on the strategies and reasoning of others.
Pedagogical questions: • If the sharing is happening part-way through the problem solving process: - Would you like to change your mind and try something different? • If the sharing is happening at the end of the problem solving process: - Would you use a different strategy next time? - How efficient was this strategy? - Which was easiest for you to understand? - What did you like about…? - What would you do differently now?	**Pedagogical questions:** • If the sharing is happening part-way through the problem solving process: - Would you like to change your mind and try something different? • If the sharing is happening at the end of the problem solving process: - Would you use a different strategy next time? - How efficient was this strategy? - How reliable was this strategy? - Which was easiest for you to understand? - What did you like about...? - What would you do differently now? - How reasonable/realistic is your answer?	**Pedagogical questions:** • If the sharing is happening part-way through the problem solving process: - Would you like to change your mind and try something different? • If the sharing is happening at the end of the problem solving process: - Would you use a different strategy next time? - How efficient was this strategy? - How reliable was this strategy? - Which was easiest for you to understand? - What did you like about...? - What would you do differently now? - How reasonable/realistic is your answer?	**Pedagogical questions:** • If the sharing is happening part-way through the problem solving process: - Would you like to change your mind and try something different? • If the sharing is happening at the end of the problem solving process: - Would you use a different strategy next time? - How efficient was this strategy? - How reliable was this strategy? - How elegant was the strategy? - Which was easiest for you to understand? - What did you like about…? - What would you do differently now? - How reasonable is your answer? - Were you expecting an answer in that range?	**Pedagogical questions:** • If the sharing is happening part-way through the problem solving process: - Would you like to change your mind and try something different? • If the sharing is happening at the end of the problem solving process: - Would you use a different strategy next time? - Which was easiest for you to understand? - What did you like about...? - What would you do differently now? - How reasonable is your answer? - Were you expecting an answer in that range?

Problem solving: Years F–2

Students benefit from working in a problem solving context in many aspects of the curriculum.

Examples

Teachers: Use your creative story telling skills to embellish these facts:

Matt and Jane have just finished playing a game. The winner is the person who has the highest score. The score is determined by the value of the counters they have collected. There are three colours of counters, worth 2, 5 and 10. Give the characters Matt and Jane a selection of counters and ask, "Who won?".

Extension Question 1: Matt thinks his counters add up to 40 points. Is that possible? Prove it!

Extension Question 2: Jane collects 5 counters. What is the highest and lowest score that Jane could have? What is the second highest/lowest score that Jane could have?

This problem facilitates a composite class working on the same problem because it has multiple entry points. It is possible to be successful in finding a solution to this problem through using: a simple counting strategy; skip counting by 2, 5 and 10; and addition strategies or multiplicative thinking.

Are there ways of arranging collections of counters that make it easier to see (at a glance) how many counters are there?

This offers the opportunity to investigate part-part-whole and arrays.

Notice a similar question in understanding and fluency. The problem solving question gives the student the opportunity to establish the idea that arrangement does matter. The fluency question gives students the opportunity to show that they have used an appropriate strategy. The understandings question gives students the opportunity to show that they appreciate that there are different possible strategies that all lead to the same solution. Notice the similarity in this problem solving question from Foundation to year 10.

NB: Problem solving questions can be detailed, but they can also be very brief.

Cathy says that she can make 27 in lots of different ways using tens and ones. What do you think?

Problem solving: Years 3–4

Students benefit from working in a problem solving context in many aspects of the curriculum.

Examples

Teachers: Use your creative story telling skills to embellish these facts:

Matt and Jane have just finished playing a game. The winner is the person who has the highest score. The score is determined by the value of the counters they have collected. There are three colours of counters, worth 3, 4 and 6 (Use an appropriate combination up to 10 x 10). Give the characters Matt and Jane a selection of counters and ask, "Who won?".

Extension Question 1: Matt thinks his counters add up to 40 (or some suitable number) points. Is that possible? Prove it.

Extension Question 2: Jane wonders if each of her counters has an even number value. Is it possible for her total score to be an odd number? What if one of the counters has an odd number value and two counters
have an even number value. Now is it possible? How? Prove it!

This problem facilitates a composite class working on the same problem because it has multiple entry points. The values of the counters can easily be changed and children can be involved in selecting values that they feel are appropriate for them.

It is possible to be successful in finding a solution to this problem through using: skip counting strategies, additive or multiplicative thinking. Notice the similarity in this problem solving question from Foundation to year 10.

Which is the greater amount of time. 2 days 7 hours and 10 minutes OR fifty two and a half hours? Estimate first. What's your 'first thinking' about this? Why? Take a class vote about which is greater. Prove it!

Problem solving questions can be detailed, but they can also be very brief.

Using the language of 'first thinking', implies that more thinking will be done and you may well change your mind.

Keeping a record of changing thoughts is an important part of students being able to observe HOW they learn. Some students find it difficult to keep a record of ideas that they no longer believe to be true, preferring to erase their initial thoughts. If the teacher makes it clear that they are marking the students THINKING, not their final answer, then erasing their changing ideas is erasing the part that the teacher wants to see evidence of. Congratulating students for changing their mind in light of new information, rather than just congratulating them when they get to an answer, will help to build a disposition of sharing ideas.

Problem solving: Years 5–6

Students benefit from working in a problem solving context in many aspects of the curriculum.

Examples

Teachers: Use your creative story telling skills to embellish these facts:

Matt and Jane are playing a computer game. In the first part of the game they get 2 minutes to collect credits that they can use later on in the game. There are three items that they can collect to earn credits, worth 14, 25 and 36 points (use an appropriate combination).

Give the characters Matt and Jane a selection of items and ask, "Who has collected the most credits?" Estimate first. What's your first thinking about this? Why? Take a class vote. Prove it!

Extension Question 1: The second time he played, Matt thought his credits added up to 400 (or some suitable number). Is that possible? Prove it!

Extension Question 2: Jane wonders if each of her credits has an even number value is it possible for her total score to be an odd number? What if one of the credits has an odd number value and two credits
have an even number value. Now is it possible? How? Prove it!

This problem facilitates a composite class working on the same problem because it has multiple entry points. The values of the credits can easily be changed and children can be involved in selecting values that they feel are appropriate for them.

It is possible to be successful in finding a solution to this problem through using: additive or multiplicative thinking. Notice the similarity in this problem solving question from Foundation to year 10.

Look at the following calculations:

4 x 2 + 2

2 + 2 x 4

If you use the same values and operations, but in a different order, do you always get the same answer? Investigate.

How could you work out the value of 15 x 16 without using the algorithm?

Notice the use of this question in each proficiency.

It is important for students to first experience questions such as this in a problem solving context, where there is no prior knowledge of a process that would work. Hence, exploration of possible approaches and checking of the validity of the solution is demanded.

As with all problem solving questions, this question gives students opportunities to reason and hence build understanding.

Problem solving questions can be detailed, but they can also be very brief. Using the language of 'first thinking', implies that more thinking will be done and you may well change your mind. Keeping a record of changing thoughts is an important part of students being able to observe HOW they learn. Some students find it difficult to keep a record of ideas that they no longer believe to be true, preferring to erase their initial thoughts. If the teacher makes it clear that they are marking the students THINKING, not their final answer, then erasing their changing ideas is erasing the part that the teacher wants to see evidence of. Congratulating students for changing their mind in light of new information, rather than just congratulating them when they reach an answer, will help to build a disposition of sharing ideas.

Problem solving: Years 7–8

Students benefit from working in a problem solving context in many aspects of the curriculum.

Examples

Teachers: Use your creative story telling skills to embellish these facts:

Matt and Jane are playing a computer game. In the first part of the game they get 2 minutes to collect credits that they can use later on in the game. There are three items that they can collect to earn credits, worth 1/2, 1/3 and 1/4 (use fractions as appropriate -these need not be unit fractions). Give the characters Matt and Jane a selection of items and ask, "Who has collected the most credits?" Estimate first.

What's your first thinking about this? Why? Take a class vote. Prove it!

Extension Question 1: On a second game, Matt thinks his credits add up to 12 ½ (or some suitable number, related to the number of credits that you say he has collected). Is that possible? Prove it!

Extension Question 2: Jane wonders if she has an even number of each of the credits, is it possible to end up with a whole number credit total?

This problem facilitates a composite class working on the same problem because it has multiple entry points. The values of the credits can easily be changed and children can be involved in selecting values that they feel are appropriate for them.

It is possible to be successful in finding a solution to this problem through using: pictures or equipment, fraction counting sequences, fraction addition or multiplication of fractions. Notice the similarity in this problem solving question from Foundation to year 10.

Martin says that it is possible to order the following equations from most to least steep. Using digital technologies or otherwise investigate this statement.

y = 2x + 3 y = 3x +2 y = x + 5

Notice the connection to this concept in each of the proficiencies.

It would be appropriate for students in year 7 to be investigating this concept, but it would be appropriate for students in year 8 to be fluent with plotting graphs of linear equations.

Lyn says that it is possible to do one simple addition sum to calculate the answer to questions such as:

11002 - 10992 892 - 882

32502 - 32492

Is there some truth in this? What do you think? Prove it!

Notice a question related to this in the reasoning proficiency. The reasoning involved in each question is the same, but the problem solving question needs to begin with strategy development. The reasoning question gives the students a structure to work with.

Problem solving questions can be detailed, but they can also be very brief. Using the language of 'first thinking', implies that more thinking will be done and you may well change your mind. Keeping a record of changing thoughts is an important part of students being able to observe HOW they learn. Some students find it difficult to keep a record of ideas that they no longer believe to be true, preferring to erase their initial thoughts. If the teacher makes it clear that they are marking the students THINKING, not their final answer, then erasing their changing ideas is erasing the part that the teacher wants to see evidence of. Congratulating students for changing their mind in light of new information, rather than just congratulating them when they get to an answer, will help to build a disposition of sharing ideas.

Problem solving: Years 9–10

Students benefit from working in a problem solving context in many aspects of the curriculum.

Examples

Teachers: Use your creative story telling skills to embellish these facts:

Matt and Jane are playing a computer game. In the first part of the game they get 2 minutes to collect credits that they can use later on in the game. There are three coloured items that they can collect to earn credits, green, red and blue, worth 1/2, 1/3 and 2/3 respectively.

Jane collected 25 credits.

Matt has eight blue credits and one more red credit than green.

If Matt collected 30 red items, is his total greater than Jane's?

What is the minimum number of red items that Matt can collect if his total is to be greater than Jane's?

Estimate first. What's your first thinking about this? Why? Take a class vote. Prove it!

Extension Question 1: In a second game, Matt thinks his credits add up to 12½ (or some suitable number, related to the number of credits that you say he has collected). Is that possible? Prove it!

Extension Question 2: Jane wonders if she has an even number of each of the credits, is it possible to end up with a whole number credit total?

This problem facilitates a composite class working on the same problem because it has multiple entry points. The values of the credits can easily be changed and students can be involved in selecting values that they feel are appropriate for them.

It is possible to be successful in finding a solution to this problem through using: pictures or equipment, fraction counting sequences, fraction addition or multiplication of fractions or algebraic modelling. Notice the similarity in this problem solving question from Foundation to year 10.

Deb says that it is possible to order the following equations from most to least steep.

Using digital technologies or otherwise investigate this statement.

y = 2x + 3

3y = 3x +2

x = y + 5

Problem solving questions can be detailed, but they can also be very brief. Using the language of 'first thinking', implies that more thinking will be done and you may well change your mind. Keeping a record of changing thoughts is an important part of students being able to observe HOW they learn. Some students find it difficult to keep a record of ideas that they no longer believe to be true, preferring to erase their initial thoughts. If the teacher makes it clear that they are marking the students THINKING, not their final answer, then erasing their changing ideas is erasing the part that the teacher wants to see evidence of. Congratulating students for changing their mind in light of new information, rather than just congratulating them when they get to an answer, will help to build a disposition of sharing ideas.

The BitL tool – mathematics years F–10

Understanding: Years F–2

What patterns/connections/relationships can you see?

This is about noticing the characteristics of familiar shapes, objects, quantities and patterns that show similarity and difference, then using these characteristics to sort and order quantities, shapes and objects. It is about looking for patterns in everything- looking for patterns in number, in shape and in data.

Pedagogical questions:

- How are these... (values/number sentences/shapes) the same as each other?
- How are these... (values/number sentences/shapes) different to each other?
- What is the connection between…?
- Which is the odd one out?
- What if... (change something), is it still…?
- Which is greater/bigger/larger/taller?*
- Which is less/smaller/shorter?*

** Asking closed questions such as these can allow the teacher to see the connections that the student is/is not making, even if the student can't articulate the connections. These questions can help the teacher to identify the root of the misconception.*

Examples

Change the object or the physical arrangement, but keep the quantity the same (eg 5 marbles/5 basketballs) and vice versa. What's the same/different?

Use 10 flip tiles; 6 orange, 4 grey. What if... (turn an orange to grey) is it still 10?

When we ask this type of question, we need to observe if the student re calculates or if they build on their understanding.

Count 6 beads into a container. Shake and tip out, asking how many will there be now?

What if I rotate a square... (sides not horizontal and vertical). Is it still a square?

Establish that a square is still a square when it is rotated such that the sides are not horizontal and vertical. Collect photographs/images of squares and rectangles in your environment. Identify that when we see these shapes in our environment, most often sides are horizontal and vertical. Why is this? Can you find examples in your environment where the sides are not horizontal and vertical?

Understanding: Years 3–4

What patterns/connections/relationships can you see?

This is about noticing and using the characteristics of shapes, objects, quantities and patterns that show similarity and difference. It is about looking for patterns and connections in number, in shape, and in data.

As students move from year 3 to year 6 we support them to make generalisations (detailed in the reasoning proficiency) from the patterns that they notice.

Noticing similarity and difference helps students to build conceptual understanding.

Pedagogical questions:

- How are these... (values/shapes/angles/questions/graphs/words/number sentences) the same as each other?
- How are these... (values/shapes/angles/questions/graphs/words/number sentences) different to each other?
- What's the connection between…?
- Which is the odd one out?
- What if...(change something), is it still…?
- Which is greater/bigger/larger/taller?*
- Which is less/smaller/shorter?*

** Asking closed questions such as these can allow the teacher to see the connections that the student is/is not making, even if the student can't articulate the connections. These questions can help the teacher to identify the root of the misconception.*

Examples

How are 4 and 6 the same as each other? How are 5 and 3 the same as each other? How is the first pair (4 and 6) different to the second pair (5 and 3)?

NB: this is more than just naming numbers as odd and even. This is explaining the characteristics of odd and even.

How are these number sequences the same as each other? How are they different to each other? Describe a rule for generating each of these number sequences. How are your rules the same as each other? How are they different?

3 , 6 , 9 , 12 , 15…
4, 7, 10, 13, 16…
1½ , 4½, 7½, 10½, 13½…
2.5, 5.5, 8.5, 11.5, 14.5…

You could provide students with the structure; 'Start with…and add on…' if they need support. Obviously in this case the numbers that the sequences start with are all different, but the amount that is added on is always 3. Questions like these help students to see that add 3 sequences, for example, can look very different to each other.

What's the connection between these two number sequences?
2,4,6,8,10 and 1,3,5,7,9

There is more than one possible answer. They both go up in twos, the last number is 8 more than the first number, each number in the second sequence is one less than its partner in the first number sequence. Notice that this question is just another way of getting children to identify and describe similarities and differences.

What if I change the sum
34 + 59 to 33 + 60… is it still the same?

Understanding: Years 5–6

What patterns/connections/relationships can you see?

This is about noticing and using the characteristics of shapes, objects, quantities and patterns that show similarity and difference. It is about looking for patterns and connections in number, in shape, and in data.

As students move from year 3 to year 6 we support them to make generalisations (detailed in the reasoning proficiency) from the patterns that they notice.

Noticing similarity and difference helps students to build conceptual understanding.

Pedagogical questions:

- How are these... (values/shapes/angles/questions/graphs/words/number sentences) the same as each other?
- How are these... (values/shapes/angles questions/graphs/words/number sentences) different to each other?
- What's the connection between…?
- Which is the odd one out?
- What if... (change something), is it still…?
- Is it always the same? Are there any exceptions?
- Estimate…
- Which is greater/bigger/larger/taller?*
- Which is less/smaller/shorter?*

** Asking closed questions such as these can allow the teacher to see the connections that the student is/is not making, even if the student can't articulate the connections. These questions can help the teacher to identify the root of the misconception.*

Examples

How is 3 x 4 the same as 4 x 3 and how is 3 x 4 different to 4 x 3?

What's the connection between these calculations?
7 x 6 = 42
3½ x 12 = 42

(The answer is not just that they both have an answer of 42!)

How are these number sequences the same as each other? How are they different to each other? Describe a rule for generating each of these number sequences. How are your rules the same as each other? How are they different?

3½, 6½, 9½, 12½, 15½…
4.1, 7.1, 10.1, 13.1, 16.1…
1¾, 4¾ , 7¾, 10¾, 13¾…
2.25, 5.25, 8.25, 11.25, 14.25…

You could provide students with the structure; 'Start with… and add on…' if they need support.

12 has an even amount of factors. 15 has an even amount of factors. 12 is an even number, 15 is an odd number. Will all numbers have an even amount of factors?

Understanding: Years 7–8

What patterns/connections/relationships can you see?

This is about noticing and using characteristics in number, algebra, measurement, geometry, probability and data.

This is about representing the patterns/relationships/rules in abstract ways, using variables.It is about identifying relationships so that we are able to reason (see reasoning: inference and generalisation) and make predictions.

Noticing similarity and difference helps students to build conceptual understanding.

Pedagogical questions:

- How are these... (values/shapes/angles/questions/graphs/words/expressions/equations) the same as each other?
- How are these...(values/shapes/angles questions/graphs/words/expressions/equations) different to each other?
- What is the connection between…?
- Which is the odd one out?
- What if... (change something), is it still…?
- Estimate…
- Which is greater/bigger/larger/taller/steeper?*
- Which is less/smaller/shorter/shallower?*

** Asking closed questions such as these can allow the teacher to see the connections that the student is/is not making, even if the student can't articulate the connections. These questions can help the teacher to identify the root of the misconception.*

Examples

How are these expressions the same as each other?
3a - b   3a + -b   3a + -1b

What's the connection between the graphs of:
y = 2x and y = 2x +1?

What's the same about the equations? What's the same about the graphs? What's different about the equations? What's different about the graphs?

Connect to reasoning: Inference
Now that you have noticed these connections, what can you infer?

Connect to problem solving: Model and plan
How can you test your idea?

Connect to reasoning: Generalisation
Generalise your findings.

Notice the connection to this concept in each of the proficiencies. This question sequence demonstrates the interconnected nature of the proficiencies.

Understanding: Years 9–10

What patterns/connections/relationships can you see?

This is about noticing and using characteristics in number, algebra, measurement, geometry, probability and data.

This is about representing the patterns/relationships/rules in abstract ways, using variables.It is about identifying relationships so that we are able to reason (see reasoning: inference and generalisation) and make predictions.

Noticing similarity and difference helps students to build conceptual understanding.

Pedagogical questions:

- How are these... (values/shapes/angles/questions/graphs/words/expressions/equations) the same as each other?
- How are these... (values/shapes/angles questions/graphs/words/expressions/equations) different to each other?
- What is the connection between…?
- Which is the odd one out?
- What if... (change something), is it still…?
- Estimate…
- Which is greater/bigger/larger/taller/steeper?*
- Which is less/smaller/shorter/shallower?*

** Asking closed questions such as these can allow the teacher to see the connections that the student is/is not making, even if the student can't articulate the connections. These questions can help the teacher identify the root of the misconception.*

Examples

What's the connection between :
36 x 34 and (x + 6) (x + 4)?
How could you use this thinking to multiply out these brackets?

Hint: The area method for multiplication of numerical values can be applied to multiplication of algebraic expressions.

Using a graphics calculator, draw the graphs of:
$y = x + 2$ and $y = x^2 + 2$

What's the same about the equations? What's the same about the graphs? What's different about the equations? What's different about the graphs? How does this connect?

Connect to reasoning: Inference
Now that you have noticed these connections, what can you infer?

Connect to problem solving: Model and plan
How can you test your idea?

Connect to reasoning: Generalisation
Generalise your findings.

Notice the connection to this concept in each of the proficiencies. This question sequence demonstrates the interconnected nature of the proficiencies.

The BitL tool – mathematics years F–10

Understanding: Years F–2	Understanding: Years 3–4	Understanding: Years 5–6	Understanding: Years 7–8	Understanding: Years 9–10
Can you answer backwards/inverse questions? This is about working flexibly with a concept.	**Can you answer backwards/inverse questions?** This is about working flexibly with a concept.	**Can you answer backwards/inverse questions?** This is about working flexibly with a concept.	**Can you answer backwards/inverse questions?** This is about working flexibly with a concept.	**Can you answer backwards/inverse questions?** This is about working flexibly with a concept.
Pedagogical questions: • If the answer is… what might the question have been? • What's missing in this number sentence/from this group/in this pattern?	**Pedagogical questions:** • If the answer is… what might the question have been? • What's missing in this number sentence/from this group/ in this pattern?	**Pedagogical questions:** • If the answer is… what might the question have been? • What's missing in this number sentence/from this group/ in this pattern?	**Pedagogical questions:** • If the answer is… what might the question have been? • What's missing in this number sentence/from this group/ in this pattern?	**Pedagogical questions:** • If the answer is… what might the question have been? • What's missing in this number sentence/from this group/ in this pattern?
Examples I'm thinking of an addition sum and the answer to my sum is 10. What might the questions have been? I need 10 children to help me, but so far we only have 6. How many more volunteers do I need? The answer is 'A SQUARE'. What might the question have been? *You could use puppets for this activity. EG. Today Mr Maths (puppet) can only say 'A square'. What question can you ask him so that he can answer you?*	**Examples** I'm thinking of a multiplication sum and the answer to my sum is 12. What might the questions have been? In ten minutes time it will be 3 o'clock. What time is it now? Extension: In ten minutes my watch will show 3 o'clock, but I know that my watch is running 5 minutes too fast. What time is it now? *This extension question is still a 'flip or backwards' style question , but a slight change requires a higher level of reasoning. Notice that the content has remained the same, but the thinking has been extended.* If the answer is 12. What might the question have been? Is it possible to use each of the four operations in the question.	**Examples** 23 x = 1311 34 + 21 = 40 + I'm thinking of a rectangle. It's area is 24cm. The length is 6cm. What is the width? One third of a class brought lunch today. If 9 students brought lunch, how many students are in this class? *In year 5 it would be appropriate for students to investigate questions such as this in a problem solving situation in order to build understanding. Hence, notice that questions similar to this are in the problem solving proficiency.*	**Examples** $^3 = 64$ I'm thinking of a number. When my number has been divided by one third I get twenty seven. What is my number?	**Examples** A rectangle of area $10cm^2$ was enlarged to create a rectangle of area $40cm^2$. What was the scale factor of enlargement. Given the hypotenuse is 15cm and one other side length is 9cm. Use Pythagoras to calculate the length of the third side.
Can you represent or calculate in different ways? This is about representing amounts, patterns, shapes and data in different ways.	**Can you represent or calculate in different ways?** This is about representing amounts, patterns, shapes and data in different ways and calculating using different processes. This is also about finding different ways to calculate the answer to computation problems. In year 3, this would include addition, subtraction and multiplication. In year 4 this would also include division.	**Can you represent or calculate in different ways?** This is about representing amounts, patterns, shapes and data in different ways. This is also about finding different ways to calculate the answer to computation problems involving all four operations (addition, subtraction, multiplication and division).	**Can you represent or calculate in different ways?** This is about representing quantities in different ways and beginning to represent situations algebraically. This is also about finding different ways to calculate the answer to computation problems. These problems may include the use of any of the four operations with rational numbers and integers.	**Can you represent or calculate in different ways?** This is about representing quantities in different ways and manipulating algebraic expressions. This is also about finding different methods for doing the same calculation. EG different algebraic methods for solving simultaneous equations. Different processes for solving a proportional reasoning problem.
Pedagogical questions: • What is another way…? • What is another way to represent that? • What is another way to work that out? • What is another way to check that?	**Pedagogical questions:** • What is another way…? • What is another way to represent that? • What is another way to work that out? • What is another way to check that? • What is another way to do that calculation? • Rename… • Represent… in multiple ways.	**Pedagogical questions:** • What is another way…? • What is another way to represent that? • What is another way to work that out? • What is another way to check that? • What is another way to do that calculation? • Simplify… • Rename… • Represent… in multiple ways.	**Pedagogical questions:** • What is another way…? • What is another way to represent that? • What is another way to work that out? • What is another way to check that? • What is another way to do that calculation? • Simplify… • Express that in another way. • Represent that algebraically.	**Pedagogical questions:** • What is another way…? • What is another way to represent that? • What is another way to work that out? • What is another way to check that? • What is another way to do that calculation? • Simplify… • Express that in another way. • Represent that algebraically.
Examples Is there more than one way to represent five? *Five can be represented as a name, numeral or a quantity, but other senses such as sound or touch could be drawn upon in experiencing or representing an amount.* Is there more than one way to count a large amount quickly/efficiently? *Notice a similar question in problem solving and fluency. The problem solving question gives the student the opportunity to establish the idea that arrangement does matter. The fluency question gives students the opportunity to show that they have an appropriate strategy. The understandings question gives students the opportunity to show that they appreciate that there are different possible strategies, that all lead to the same solution.* Rename 156. *156 can be made from 1 hundred, 5 tens and 6 ones OR 15 tens and 6 ones etc.*	**Examples** Rename 1250. *(1250 can be made from 1 thousand, 2 hundreds and 5 tens OR 12 hundreds and 5 tens etc)* Represent ½ in multiple ways *Fractions of: a shape, a collection, an amount, a line.* Represent 36 in as many different arrays as possible. Work out 287 ÷ 7 in two (or more) different ways. *By the end of year 4 we would expect students to have a method to be able to answer questions such as this, but students benefit from having different suitable approaches, so that they can begin to choose the most appropriate approach for a particular calculation. For this question a student may reason that 7 x 10 =70 and 70 x 4 =280, so there are 40 7's in 280 and so there would be 41 7's in 287.*	**Examples** Is there more than one way to work out 15 x 16. *Notice the use of this question in each proficiency.* Which of the following fractions can you simplify? 18/30 18/29 13/52 7/21 How would you represent 'evens chance' as a fraction, decimal and percentage?	**Examples** Calculate the average of this data set in three different ways: 103, 79, 15, 89, 94 *See connection to reasoning.* Cindy calculated 11% of \$80, by calculating 10% of \$80, then 1% of \$80 and adding them. What is another way to do that calculation? I'm thinking of a number. I double it and subtracted 6. The answer is 36. Represent that algebraically.	**Examples** What are some other ways to express: $6^5 \div 6^3$ Simplify. Given a symmetrical trapezium. What are three different ways that the area could be calculated?

The BitL tool – mathematics years F–10

Reasoning: Years F–2	Reasoning: Years 3–4	Reasoning: Years 5–6	Reasoning: Years 7–8	Reasoning: Years 9–10
In what ways can you prove…? This is about convincing yourself and others about your mathematical thinking. At this stage proof would involve using equipment, drawings and simple calculations. It is important to evaluate different ways of proving the same idea and justify the choices that are made.	**In what ways can you prove…?** This is about convincing yourself and others about your mathematical thinking. At this stage proof would involve using equipment, diagrams and simple calculations. It is important to evaluate different ways of proving the same idea and justify the choices that are made.	**In what ways can you prove…?** This is about convincing yourself and others about your mathematical thinking. Across years 3 to 6; proof, like under-standing, will be represented using models, diagrams and symbols/calculations. The complexity of the diagrams will develop to be more abstract in appearance and will use mathematical conventions, such as those for labeling angles or communicating length. It is important to evaluate different ways of proving the same idea and justify the choices that are made.	**In what ways can you prove…?** This is about convincing yourself and others about your mathematical thinking. At this stage proof would still involve diagrams (using mathematical conventions) and calculations. It would also start to involve algebraic representation. It is important to evaluate different ways of proving the same idea and justify the choices that are made.	**In what ways can you prove…?** This is about convincing yourself and others about your mathematical thinking. At this stage proof could involve diagrams (using mathematical conventions) and calculations. It would also involve algebraic representation where appropriate. It is important to evaluate different ways of proving the same idea and justify the choices that are made.
Pedagogical questions: • Prove that… • Convince me, yourself or someone who thinks differently… • Try not to ask me IF you are correct, but instead try to tell when you KNOW that you are correct. Then share how you know. • What else could it be?	**Pedagogical questions:** • Prove that… • Convince me, yourself, someone who thinks differently… • Try not to ask IF you are correct, but instead try to tell when you KNOW that you are correct. Then share HOW you know. • What else could it be?	**Pedagogical questions:** • Prove that… • Convince me, yourself, someone who thinks differently… • Don't ask IF you are correct; tell when you KNOW you're correct,and tell HOW you know. • What else could it be? • Why is that the best way to show…?	**Pedagogical questions:** • Prove that… • Convince me, yourself, someone who thinks differently… • Don't ask IF you are correct; tell when you KNOW you're correct,and tell HOW you know. • What else could it be? • Why is that the best way to show…?	**Pedagogical questions:** • Prove that… • Convince me, yourself, someone who thinks differently… • Don't ask IF you are correct; tell when you KNOW you're correct,and tell HOW you know. • What else could it be? • Why is that the best way to show…?
Examples Prove that one ten and seventeen ones is worth the same as two tens and seven ones. Convince me/convince yourself!	**Examples** *Connected to the problem solving question:* '134 can be made from 1 hundred, 3 tens and 4 ones. If you only had tens and ones, could you still build 134?' *The reasoning (proof) emphasis would be:* Prove that your different solutions are worth the same as each other.	**Examples** Meg says that 14 x 17 will have the same answer as 15 x 16. Why do you think that Meg has made this connection? Do you agree/disagree? Prove it!	**Examples** Convince me that the angle sum of any triangle is 180 degrees. Use this to show the angle sum of any quadrilateral.	**Examples** Show that the area of a trapezium with parallel sides of length 'a' and 'b' and perpendicular height 'h', will have an area of: ½ (a + b) h
In what ways can you communicate? This is about making thinking visible. At this stage of development it will often be achieved using simple mathematical language (spoken or written) and drawings. It is important to evaluate different ways to communicate the same idea.	**In what ways can you communicate?** This is about making thinking visible and communicating a logical progression of ideas. It is important to evaluate different ways to communicate the same idea.	**In what ways can you communicate?** This is about making thinking visible and sharing your ideas using mathematical terminology, diagrams and symbolic representations. It is important to evaluate different ways to communicate the same idea.	**In what ways can you communicate?** This is about making thinking visible and sharing your ideas using mathematical terminology, diagrams and symbolic representations (including simple algebraic representations). It is important to evaluate different ways to communicate the same idea.	**In what ways can you communicate?** This is about making thinking visible and sharing your ideas using mathematical terminology, diagrams and symbolic representations (including algebraic representations). It is important to evaluate different ways to communicate the same idea.
Pedagogical questions: • How come…? • Explain it/why? (To a peer) • Can you show me how that works? • Why did you choose to…? • Why is it not… (followed by an incorrect name or process)? • Why can't I… (followed by an incorrect name or process)?	**Pedagogical questions:** • What's the best way to record your results and why? • How come…? • Explain it/why? (to somebody who hasn't been involved in the learning, eg parent, a child in a different class). • Can you show me how that works? • Why did you choose to…? • Why is it not… (followed by an incorrect name or process)? • Why can't I… (followed by an incorrect name or process)?	**Pedagogical questions:** • What's the best way to record your results and why? • How come…? • Explain it/why? (to somebody who hasn't been involved in the learning) • Can you show me how that works? • Why did you choose to…? • Why is it not… (followed by an incorrect name or process)? • Why can't I… (followed by an incorrect name or process)?	**Pedagogical questions:** • What's the best way to record your results and why? • How come…? • Explain it/why? (to somebody who hasn't been involved in the learning) • Can you show me how that works? • Why did you choose to…? • How does your formula show…? • Why would (a graph/a formula/ a diagram) be useful/not helpful? • Why is it not… (followed by an incorrect name or process)? • Why can't I… (followed by an incorrect name or process)?	**Pedagogical questions:** • What's the best way to record your results and why? • How come…? • Explain it/why? (to somebody who hasn't been involved in the learning) • Can you show me how that works? • Why did you choose to…? • How does your formula show…? • Why would (a graph/a formula/ a diagram) be useful/not helpful? • Why is it not… (followed by an incorrect name or process)? • Why can't I… (followed by an incorrect name or process)?
Examples Communication of mathematical ideas can be emphasised in any proficiency, with any content.	**Examples** Communication of mathematical ideas can be emphasised in any proficiency, with any content.	**Examples** Communication of mathematical ideas can be emphasised in any proficiency, with any content.	**Examples** Communication of mathematical ideas can be emphasised in any proficiency, with any content.	**Examples** Communication of mathematical ideas can be emphasized in any proficiency, with any content.

The BitL tool – mathematics years F–10

Reasoning: Years F–2	Reasoning: Years 3–4	Reasoning: Years 5–6	Reasoning: Years 7–8	Reasoning: Years 9–10
In what ways can your thinking be generalised? This is very strongly connected to looking for patterns and relationships. This is about making statements that describe a pattern that always exists. At this stage general rules are most likely to be described verbally and in every day language.	**In what ways can your thinking be generalised?** This is very strongly connected to looking for patterns and relationships. This is about making statements that describe a pattern that always exists. At this stage generalisation may be verbal or written. Across years 3 and 4, age appropriate mathematical terminology will be used increasingly.	**In what ways can your thinking be generalised?** This is very strongly connected to looking for patterns and relationships. This is about making statements that describe a pattern that always exists. At this stage, generalisations will often be written. Across years 5 and 6, age appropriate mathematical terminology will be used. Generalisations are certainties. They will always be true and it is not necessary to collect further information.	**In what ways can your thinking be generalised?** This is very strongly connected to looking for patterns and relationships. This is about making statements that describe a pattern that always exists. At this stage generalisations will be written. Across years 7 and 8, ageappropriate mathematical terminology and conventions will be used. Generalisations will start to be expressed algebraically. Generalisations are certainties. They will always be true and it is not necessary to collect further information.	**In what ways can your thinking be generalised?** This is very strongly connected to looking for patterns and relationships. This is about making statements that describe a pattern that always exists. At this stage, generalisations will be written. Across years 9 and 10, age appropriate mathematical terminology and conventions will be used. Generalisations will be expressed algebraically where appropriate. Generalisations are certainties. They will always be true and it is not necessary to collect further information.
Pedagogical questions: • Why are these always the same/different? • Is there a rule that we could use to describe…? • Is there a rule that always works? • What makes these the same?	**Pedagogical questions:** • Why are these always the same/different? • Is there a rule that we could use to describe…? • Is there a rule that always works? • What makes these different processes the same?	**Pedagogical questions:** • Why are these always the same/different? • Is there a rule that we could use to describe…? • Is there a rule that always works? • What makes these different processes the same? • Use sentence structures, such as the following, as writing frames to encourage generalisation: – The bigger/smaller the… – The older/younger the…	**Pedagogical questions:** • Why are these always the same/different? • Is there a rule that we could use to describe...? • Is there a rule that always works? • What makes these different processes the same? • Express that algebraically. • Use sentence structures, such as the following, as writing frames to encourage generalisation: – The bigger/smaller the… – The older/younger the… – The further/closer the…	**Pedagogical questions:** • Why are these always the same/different? • Is there a rule that we could use to describe…? • Is there a rule that always works? • What makes these different processes the same? • Express that algebraically. • Use sentence structures, such as the following, as writing frames to encourage generalisation: – The bigger/smaller the… – The older/younger the… – The further/closer the…
Examples Sammy has found three ways to make 27 using tens and ones (MAB). Can you find these three ways? Will there always be three ways to make a number? *Of course, it's not the same for all numbers there are 4 ways for numbers in the 30's, 5 ways for numbers in the 40's etc* *General rules may be about:* *Patterns in number sequences, such as; "When you count by tens, the ones number always stays the same".* *Expanding or repeating patterns, such as; "The next group always has one more bead than the group before".* *Similarities in shapes, such as; "When you join the opposite corners of a square or a rectangle you always make triangles".*	**Examples** What is it about all odd numbers that makes them the same and all even numbers the same? So what makes the odd numbers different to the even numbers?	**Examples** Draw a rectangle and enlarge it with a scale factor of two. What do you notice about the area of the enlarged rectangle compared to the area of the original rectangle? Start with a rectangle of a different area. What do you notice about the area of the enlarged rectangle compared to the area of the original rectangle? Exchange some results with a peer. Is there a rule about how the area of the enlarged rectangle compares to the area of the original? Does this rule work if you enlarge your rectangle by a scale factor of three? *Enlargement is a year 5 content description, but investigating it in this way brings in square numbers, which is a year 6 content description.*	**Examples** You have shown the angle sum of a triangle to be 180 degrees and the angle sum of any quadrilateral to be 360 degrees. Can you make a statement about the angle sum of any polygon? Is there a rule that always works? Can you write an algebraic expression relating the number of sides that the polygon has and the total of the internal angles?	**Examples** Use repeated applications of simple interest to show compound interest of 3% p/a on an initial investment of $500 for 5 years. Could you simplify this process? What could you do instead of repeating the same calculation 5 times over? Would you process work for any amount? Express your process algebraically. Compare your thinking to a given formula for compound interest.

The BitL tool – mathematics years F–10

Reasoning: Years F–2

What can you infer?

This is about developing logical thought processes. These processes sometimes follow the structure: if…, then… This type of thinking helps to create new information from known information.

Logical thought can also be about working out a set of possibilities and narrowing them down as you get more information.

Pedagogical questions:

- Now that you know… can you work out…?
- I'm thinking of...(a number/a shape) and I'm going to give you some clues… Can you work out what my number/ shape is?
- I'm thinking of... (a number/a shape) and I'm going to give you some clues… Can you work out what the possible answers are?
- I'm thinking of... (a number/a shape). You can ask questions
  to help you to work out what it is, but I can only answer yes or no.
- You could use sentence structures such as:
  - If… then…
  - Because I know… I also know…

Examples

Following on from the understandings question:
Use 10 flip tiles 6 orange, 4 grey. What if (turn an orange to grey) is it still 10?

Now adding in the reasoning (communication) element:
Why is it still the same? How does that work?

Further reasoning (proof):
Prove that to me.

Further reasoning (generalising):
What if we start with 12 flip tiles, 6 orange, 6 grey. If we turn an orange tile over, we have 7 orange and 5 grey. Is it still 12? How is that like our last question? Why is it still the same? Will this work for any number of tiles?

I have some counters in this bag. If I tell you that here are between 10 and 18 counters. How many could it be? If I tell you that the counters can be made into two equal groups, what do you think now? (Alternatively you could say: I would say this number if I skip counted by two's, starting at 10) If I tell you that the counters could be made into groups of 4, what do you think now? Can you ask a question that would help you to decide how many counters there are in this bag? (You can't ask: "Is it 12!")

Reasoning: Years 3–4

What can you infer?

This is about developing logical thought processes. These processes sometimes follow the structure: if…, then… This type of thinking helps to create new information from known information.

Logical thought can also be about working out a set of possibilities and narrowing them down as you get more information.

Pedagogical questions:

- Now that you know… can you work out…?
- I'm thinking of... (a number/a shape) and I'm going to give you some clues… Can you work out what my number/shape is?
- I'm thinking of... (a number/a shape) and I'm going to give you some clues… Can you work out what the possible answers are?
- I'm thinking of... (a number/a shape). You can ask questions to help you
  to work out what it is, but I can only answer yes or no.
- You could use sentence structures such as:
  - If… then…
  - Because I know… I also know…

Examples

I am thinking of a number. My number is between one thousand and two thousand. It uses the digits 0, 1, 3, and 6. What could my number be? If I say that my number is odd. What could it be now? Can you ask a question that would help you to identify my number?

I'm thinking of 2 numbers. When I multiply these numbers together I get 12. When I add them together I get an even number.

x = 12
\+ = even number

What do you think my two numbers could be? Are you certain or do you want more information?

This question requires inference, but you could also emphasise the need for clear communication of thinking.

Reasoning: Years 5–6

What can you infer?

This is about developing logical thought processes. These processes sometimes follow the structure: if…, then… This type of thinking helps to create new information from known information.

Logical thought can also be about working out a set of possibilities and narrowing them down as you get more information.

We make an inference when the known information suggests a particular connection or connections. Further information/ testing may show an initial inference to be true or false.

Pedagogical questions:

- Now that you know… can you work out…?
- I'm thinking of... (a number/a shape) and I'm going to give you some clues… Can you work out what my number/shape is?
- I'm thinking of... (a number/a shape) and I'm going to give you some clues… Can you work out what the possible answers are?
- I'm thinking of... (a number/a shape). You can ask questions to help you to work out what it is, but I can only answer yes or no.
- You could use sentence structures such as:
  - If… then…
  - Because I know… I also know…

Examples

The arm spans of six people were recorded. Here are the measurements:
63, 70, 65, 68, 71, 64
What units do you think were used? What additional information would you like? What do you think if I tell you that they are people of school age?

I'm thinking of a fraction between 0 and 1. I can answer yes or no to your questions. You can't ask the same type of question more than once. Can you work out what my fractions is?

The first time you try a question like this, students will probably need support to try to think about different styles of questions that they could ask. Examples are: Is it greater than ½? Is it smaller than ¼? Is it a unit fraction? Is the denominator an odd number? Is it equivalent to 2/6? Would I need more than 6 of them to make a whole?

Reasoning: Years 7–8

What can you infer?

This is about developing logical thought processes. These processes sometimes follow the structure: if…, then… This type of thinking helps to create new information from known information.

Logical thought can also be about working out a set of possibilities and narrowing them down as you get more information.

We make an inference when the known or assumed information suggests a particular connection or connections. Further information/testing may show an initial inference to be true or false.

Pedagogical questions:

- Now that you know… can you work out…?
- I'm thinking of... (a number/a shape) and I'm going to give you some clues… Can you work out what my number/shape is?
- I'm thinking of... (a number/a shape) and I'm going to give you some clues… Can you work out what the possible answers are?
- I'm thinking of... (a number/a shape). You can ask questions to help you to work out what it is, but I can only answer yes or no.
- You could use sentence structures such as:
  - If (assumption) then…
  - Because I know… I also know…

Examples

Try these questions:
$102^2 – 92^2=$
$92^2 – 82^2=$
$82^2 – 72^2=$
$72^2 – 62^2=$
What do you notice about the answers to these questions?
What can you infer from these observations about the case when the difference between the square numbers is two, so that the questions become:

$102^2 – 82^2=$

$92^2 – 72^2=$

etc

What do you think now?

Is it possible to make a generalisation? Do you need more information? How could you test your idea?

Reasoning: Years 9–10

What can you infer?

Logical thought can also be about working out a set of possibilities and narrowing them down as you get more information.

We make an inference when the known or assumed information suggests a particular connection or connections. Further information/testing may show an initial inference to be true or false.

Pedagogical questions:

- Now that you know… can you work out…?
- I'm thinking of...(a number/a shape) and I'm going to give you some clues… Can you work out what my number/shape is?
- I'm thinking of... (a number/a shape) and I'm going to give you some clues… Can you work out what the possible answers are?
- I'm thinking of... (a number/a shape). You can ask questions to help you to work out what it is, but I can only answer yes or no.
- Under what conditions would…?
- You could use sentence structures such as:
  - If (assumption) then…
  - Because I know… I also know…

Examples

When are 3a, $3a^2$ and ^{3} equal to each other?

Under what conditions is the order from smallest to largest: 3a, $3a^2$ and $3a^3$?

Under what conditions is the order from smallest to largest: $3a^3$, $3a^2$ and 3a?