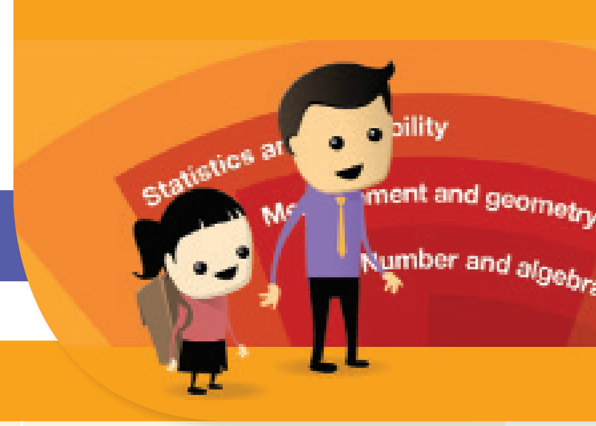
The BitL tool - mathematics years 7-8



Fluency: Years 7-8

What can you recall?

This is about remembering/identifying mathematical, names, shapes, symbols, facts, processes and formulas that are important to know when working with mathematical ideas.

Pedagogical questions:

- How could you record that mathematically?
- How could you... (eg calculate that)?
- How could you use a calculator to…?
- Can you remember a way to…?
- What is the value of… (a calculation that you would expect automatic recall of, eg times tables, some square numbers, square roots of perfect squares, some powers of 10 (eg 102=100, 103=1000)?
- What is the name of…?
- What is the symbol for…?
- What is the formula for…?
- How many…?
- How much…?

Examples

What formula can be used to calculate the area of a rectangle/a triangle?

Name the parts of a circle.

What is the value of 1/3 + 3/4?
What is the value of 3/4 - 1/3?
What is the value of 1/3 x 3/4?
What is the value of 3/4 ÷ 1/3?

Can you choose and use your mathemtics flexibly?

To be able to choose and use mathematics efficiently students need to be able to recall processes and facts. Choosing and using is about selecting (age appropriate) processes, facts and mathematical language appropriate to the context.

Pedagogical questions:

- Choose a way to record that mathematically.
- Choose a way to (estimate/rename/measure/compare/order/calculate/partition/rearrange/regroup/record/show/represent that).
- Use mathematical language to describe…
- What would be an efficient way to... (count/measure/order/compare/add on/subtract/multiply/divide/calculate/draw/record/simplify/expand/rearrange/substitute that)?
- How could you use a calculator/computer to…?

Examples

Calculate how many centimetre squares there are in a metre square. Calculate how many metres square there are in a kilometre square.

Kym was driving to a concert. She used ¼ of a tank of fuel to get there, but she took a different route home and used 1/3 of a tank of fuel. If the tank was full before she left home and she didn't put any additional fuel in, what fraction of a tank does Kym have left?

Factorise: 6xy + 2x

Problem solving: Years 7-8

Students benefit from working in a problem solving context in many aspects of the curriculum.

How can you interpret?

This is about creating meaning from the problem that has been presented or created by the student in response to curiosity about real world applications of mathematics that are relevant to the student.

It is useful to have the students describe (in their own words) what they have been asked to do. Descriptions of the task could be oral or written, as appropriate for the students and the task.

Pedagogical questions:

- What are you being asked to find out, demonstrate or prove?
- What information is helpful?
- What information is not useful?
- What additional information would be useful?

Closed questions can be useful to check if the student has accessed the information given in the question, for example

-How many…?

-How much…?

-When…?

(These questions will vary depending on the context of the problem)

In what ways can you model and plan?

This is about describing a problem mathematically. Across years 7 to 10, ideas are represented using models, diagrams and symbols. There is an increasing emphasis on abstract symbolic representation.

It is important for students to think about how they will attempt to solve the problem, rather than rushing into taking measurements or making calculations without first thinking about how helpful that will be.

Pedagogical questions:

- Do you have an idea?
- What could you try?
- Have you done a problem like this one before?
- How could you test your idea?
- How might you start?
- Can you represent the problem as a picture or by using equipment?
- Can you represent the information numerically or symbolically?
- What questions could you ask (to find that out)?
- What information could you put in a diagram to support your thinking?
- What strategies have you used in the past when you have been stuck?
- Speak to a peer. Ask them to show you what they are trying.

In what ways can you solve and check?

This is the mechanics of problem solving-the doing of calculations and checking how appropriate the answer.

Pedagogical questions:

- How can you calculate that?
- What processes could you try?
- Does that seem right to you?
- How can you check your answer?
- Do other people think that too?

Reflect

Students need to reflect on how reasonable their solution is. They should consider if they have made an appropriate interpretation in relation to the context of the problem.

There are different ways to solve problems and different ways to explain your thinking. At every stage of development, students benefit from sharing and reflecting on the strategies and reasoning of others.

Pedagogical questions:

- If the sharing is happening part-way through the problem solving process:
  - Would you like to change your mind and try something different?
- If the sharing is happening at the end of the problem solving process:
  - Would you use a different strategy next time?
  - How efficient was this strategy?
  - How reliable was this strategy?
  - How elegant was the strategy?
  - Which was easiest for you to understand?
  - What did you like about…?
  - What would you do differently now?
  - How reasonable is your answer?
  - Were you expecting an answer in that range?

Examples

Teachers: Use your creative story telling skills to embellish these facts:

Matt and Jane are playing a computer game. In the first part of the game they get 2 minutes to collect credits that they can use later on in the game. There are three items that they can collect to earn credits, worth 1/2, 1/3 and 1/4 (use fractions as appropriate -these need not be unit fractions). Give the characters Matt and Jane a selection of items and ask, "Who has collected the most credits?" Estimate first. What's your first thinking about this? Why? Take a class vote. Prove it!

Extension Question 1: On a second game, Matt thinks his credits add up to 12 ½ (or some suitable number, related to the number of credits that you say he has collected). Is that possible? Prove it!

Extension Question 2: Jane wonders if she has an even number of each of the credits, is it possible to end up with a whole number credit total?

This problem facilitates a composite class working on the same problem because it has multiple entry points. The values of the credits can easily be changed and children can be involved in selecting values that they feel are appropriate for them.

It is possible to be successful in finding a solution to this problem through using: pictures or equipment, fraction counting sequences, fraction addition or multiplication of fractions. Notice the similarity in this problem solving question from Foundation to year 10.

Martin says that it is possible to order the following equations from most to least steep. Using digital technologies or otherwise investigate this statement.

y = 2x + 3 y = 3x +2 y = x + 5

Notice the connection to this concept in each of the proficiencies.

It would be appropriate for students in year 7 to be investigating this concept, but it would be appropriate for students in year 8 to be fluent with plotting graphs of linear equations.

Lyn says that it is possible to do one simple addition sum to calculate the answer to questions such as:

11002 - 10992 892 - 882 32502 - 32492

Is there some truth in this? What do you think? Prove it!

Notice a question related to this in the reasoning proficiency. The reasoning involved in each question is the same, but the problem solving question needs to begin with strategy development. The reasoning question gives the students a structure to work with.

Problem solving questions can be detailed, but they can also be very brief. Using the language of 'first thinking', implies that more thinking will be done and you may well change your mind. Keeping a record of changing thoughts is an important part of students being able to observe HOW they learn. Some students find it difficult to keep a record of ideas that they no longer believe to be true, preferring to erase their initial thoughts. If the teacher makes it clear that they are marking the students THINKING, not their final answer, then erasing their changing ideas is erasing the part that the teacher wants to see evidence of. Congratulating students for changing their mind in light of new information, rather than just congratulating them when they get to an answer, will help to build a disposition of sharing ideas.

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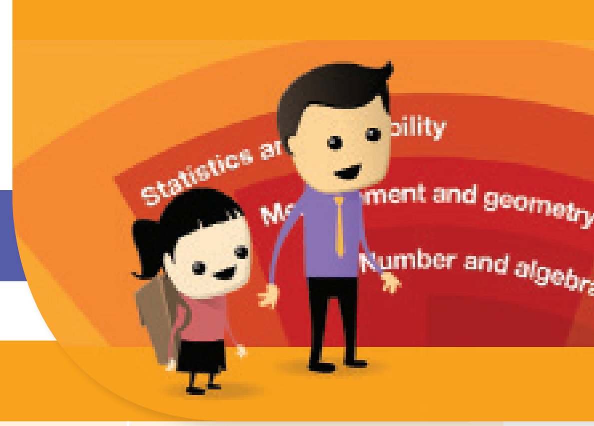

Understanding: Years 7-8

What patterns/connections/relationships can you see?

This is about noticing and using characteristics in number, algebra, measurement, geometry, probability and data.

This is about representing the patterns/relationships/rules in abstract ways, using variables.It is about identifying relationships so that we are able to reason (see reasoning: inference and generalisation) and make predictions.

Noticing similarity and difference helps students to build conceptual understanding.

Pedagogical questions:

- How are these... (values/shapes/angles/questions/graphs/words/expressions/equations) the same as each other?
- How are these...(values/shapes/angles questions/graphs/words/expressions/equations) different to each other?
- What is the connection between…?
- Which is the odd one out?
- What if... (change something), is it still…?
- Estimate…
- Which is greater/bigger/larger/taller/steeper?*
- Which is less/smaller/shorter/shallower?*

** Asking closed questions such as these can allow the teacher to see the connections that the student is/is not making, even if the student can't articulate the connections. These questions can help the teacher to identify the root of the misconception.*

Examples

How are these expressions the same as each other?

3a - b   3a + -b   3a + -1b

What's the connection between the graphs of: y = 2x and y = 2x +1?

What's the same about the equations? What's the same about the graphs? What's different about the equations? What's different about the graphs?

Connect to reasoning: Inference

Now that you have noticed these connections, what can you infer?

Connect to problem solving: Model and plan

How can you test your idea?

Connect to reasoning: Generalisation

Generalise your findings.

Notice the connection to this concept in each of the proficiencies. This question sequence demonstrates the interconnected nature of the proficiencies.

Can you answer backwards/ inverse questions?

This is about working flexibly with a concept.

Pedagogical questions:

- If the answer is… what might the question have been?
- What's missing in this number sentence/from this group/in this pattern?

Examples

$^3 = 64$

I'm thinking of a number. When my number has been divided by one third I get twenty seven. What is my number?

Can you represent or calculate in different ways?

This is about representing quantities in different ways and beginning to represent situations algebraically. This is also about finding different ways to calculate the answer to computation problems. These problems may include the use of any of the four operations with rational numbers and integers.

Pedagogical questions:

- What is another way…?
- What is another way to represent that?
- What is another way to work that out?
- What is another way to check that?
- What is another way to do that calculation?
- Simplify…
- Express that in another way.
- Represent that algebraically.

Examples

Calculate the average of this data set in three different ways:

103, 79, 15, 89, 94

See connection to reasoning.

Cindy calculated 11% of $80, by calculating 10% of $80, then 1% of $80 and adding them. What is another way to do that calculation?

I'm thinking of a number. I doubled it and subtracted 6. The answer is 36. Represent that algebraically.

Reasoning: Years 7-8

In what ways can you prove…?

This is really about convincing yourself and others about your mathematical thinking.

At this stage proof would still involve diagrams (using mathematical conventions) and calculations. It would also start to involve algebraic representation.

It is important to evaluate different ways of proving the same idea and justify the choices that are made.

Pedagogical questions:

- Prove that…
- Convince me, yourself, someone who thinks differently...
- Don't ask me IF you are correct, tell me when you KNOW you're correct, and tell me HOW you know.
- What else could it be?
- Why is that the best way to show…?

Examples

Convince me that the angle sum of any triangle is 180 degrees. Use this to show the angle sum of any quadrilateral.

In what ways can you communicate?

This is about making thinking visible and sharing your ideas using mathematical terminology, diagrams and symbolic representations (including simple algebraic representations).

It is important to evaluate different ways to communicate the same idea.

Pedagogical questions:

- What's the best way to record your results and why?
- How come…?
- Explain it/why? (to somebody who hasn't been involved in the learning).
- Can you show me how that works?
- Why did you choose to…?
- How does your formula show…?
- Why would...(a graph/a formula/a diagram) be useful/not helpful?
- Why is it not… (followed by an incorrect name or process)?
- Why can't I… (followed by an incorrect name or process)?

Examples

Communication of mathematical ideas can be emphasised in any proficiency, with any content.

In what ways can your thinking be generalised?

This is very strongly connected to looking for patterns and relationships. This is about making statements that describe a pattern that always exists. At this stage generalisations will be written. Across years 7 and 8, age appropriate mathematical terminology and conventions will be used. Generalisations will start to be expressed algebraically.

Generalisations are certainties. They will always be true and it is not necessary to collect further information.

Pedagogical questions:

- Why are these always the same/different?
- Is there a rule that we could use to describe...?
- Is there a rule that always works?
- What makes these different processes the same?
- Express that algebraically.
- Use sentence structures, like the following, as writing frames to encourage generalisation:
  - The bigger/smaller the…
  - The older/younger the… the…
  - The further/closer the… the…

Examples

You have shown the angle sum of a triangle to be 180 degrees and the angle sum of any quadrilateral to be 360 degrees. Can you make a statement about the angle sum of any polygon? Is there a rule that always works? Can you write an algebraic expression relating the number of sides that the polygon has and the total of the internal angles?

What can you infer?

This is about developing logical thought processes. These processes sometimes follow the structure: if…, then… This type of thinking helps to create new information from known information.

Logical thought can also be about working out a set of possibilities and narrowing them down as you get more information.

We make an inference when the known or assumed information suggests a particular connection or connections. Further information/testing may show an initial inference to be true or false.

Pedagogical questions:

- Now that you know… can you work out…?
- I'm thinking of... (a number/a shape) and I'm going to give you some clues… Can you work out what my number/shape is?
- I'm thinking of... (a number/a shape) and I'm going to give you some clues… Can you work out what the possible answers are?
- I'm thinking of... (a number/a shape). You can ask questions to help you to work out what it is, but I can only answer yes or no.
- You could use sentence structures such as:
  - If (assumption) then…
  - Because I know… I also know…

Examples

Try these questions:

$102^2 - 92^2 =$

$92^2 - 82^2 =$

$82^2 - 72^2 =$

$72^2 - 62^2 =$

What do you notice about the answers to these questions?

What can you infer from these observations about the case when the difference between the square numbers is two, so that the questions become:

$102^2 - 82^2 =$

$92^2 - 72^2 =$

etc?

What do you think now?

Is it possible to make a generalisation? Do you need more information? How could you test your idea?