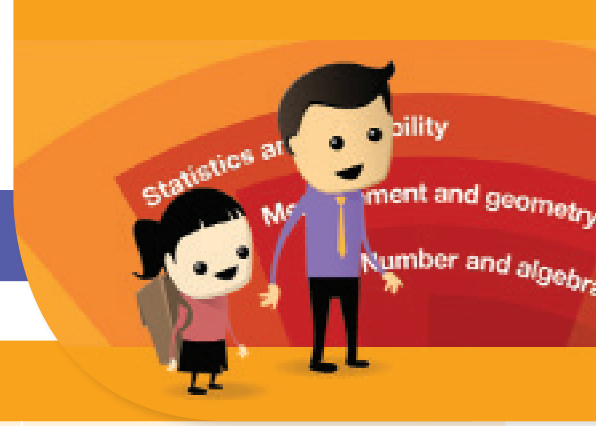
The BitL tool - mathematics years 5-6



Fluency: Years 5-6

What can you recall?

This is about remembering/identifying mathematical, names, shapes, symbols, facts, processes and formulas that are important to know when working with mathematical ideas.

Pedagogical questions:

- How could you record that mathematically?
- How could you... (eg calculate that)?
- How could you use a calculator to…?
- Can you remember a way to…?
- What is the value of… (a calculation that you would expect automatic recall of eg number pairs to 10, to 100, times tables)?
- What is the name of…?
- What is the symbol for…?
- What is the formula for…?
- How many…?
- How much…?

Examples

What metric units of measurement are used for length, area, volume, capacity and mass (including large and small increments)?

How many millimetres are there in a centimetre?
How many centimetres are there in a metre?
How many metres are there in a kilometre?

What is the name of... (show a range of 2D shapes and 3D objects, including prisms and pyramids)?

Can you remember a way to multiply two 2 digit numbers together eg 36 x 54?

Can you remember a way to divide a large number by a single digit number (with remainders) eg 2847 ÷ 7?

Can you choose and use your mathematics flexibly?

To be able to choose and use mathematics efficiently students need to be able to recall processes and facts. Choosing and using is about selecting (age appropriate) processes, facts and mathematical language appropriate to the context.

Pedagogical questions:

- Choose a way to record that mathematically.
- Choose a way to... (estimate/rename/measure/compare/order/calculate/partition/rearrange/regroup/record/show/represent) that.
- Use mathematical language to describe…
- What would be an efficient way to...(count/measure/order/compare/add on/subtract/multiply/divide/calculate/draw/record/simplify) that?
- How could you use a calculator/computer to…?

Examples

Order a selection of angles from smallest to largest. Measure and record the size of each angle to verify the order that you have placed them in.

Calculate how many millimetres there are in a kilometre. Calculate how many grams there are in a tonne.

Problem solving: Years 5-6

Students benefit from working in a problem solving context in many aspects of the curriculum.

How can you interpret?

This is about creating meaning from the problem that has been presented or created by the student in response to curiosity about their world.

It is useful to have the students describe (in their own words) what they have been asked to do. Descriptions of the task could be oral or written, as appropriate for the students and the task.

Pedagogical questions:

- What are you being asked to find out, demonstrate or prove?
- What information is helpful?
- What information is not useful?
- What additional information would be useful?

Closed questions can be useful to check if the student has accessed the information given in the question, for example
- How many…?
- How much…?
- When…?

(These questions will vary depending on the context of the problem)

In what ways can you model and plan?

This is about describing a problem mathematically. Across years 3 to 6, ideas are represented using models, pictures and symbols. The complexity of the pictures will develop from those representing an image of the problem (in years 3 and 4) to those that support thinking about the problem and are more abstract in appearance (in years 5 and 6).

It is important to think about how you will attempt to solve the problem, rather than rushing into taking measurements or making calculations without first thinking about how helpful that will be.

Pedagogical questions:

- Do you have an idea?
- What could you try?
- Have you done a problem like this one before?
- How could you test your idea?
- How might you start?
- Can you represent the problem as a picture or by using equipment?
- Can you act it out?
- Can you represent the information using numbers and symbols?
- What questions could you ask (to find that out)?
- What information could you put in a diagram to support your thinking?
- When we are being good problem solvers, what do we do to get started?
- Speak to a peer. Ask them to show you what they are trying.

In what ways can you solve and check?

This is the mechanics of problem solving - the doing of calculations (the adding/subtracting/sharing/grouping/constructing) and checking how appropriate the answer is.

Pedagogical questions:

- How can you... (add those numbers together/subtract that amount/multiply those amounts/divide those amounts)?
- What processes could you try?
- Does that seem right to you?
- How can you check your answer?
- Do other people think that too?

Reflect

Students need to reflect on how reasonable their solution is. They should consider if they have made an appropriate interpretation in relation to the context of the problem.

There are different ways to solve problems and different ways to explain your thinking. At every stage of development, students benefit from sharing and reflecting on the strategies and reasoning of others.

Pedagogical questions:

- If the sharing is happening part-way through the problem solving process:
  - Would you like to change your mind and try something different?
- If the sharing is happening at the end of the problem solving process:
  - Would you use a different strategy next time?
  - How efficient was this strategy?
  - How reliable was this strategy?
  - Which was easiest for you to understand?
  - What did you like about...?
  - What would you do differently now?
  - How reasonable/realistic is your answer?

Examples

Teachers: Use your creative story telling skills to embellish these facts:

Matt and Jane are playing a computer game. In the first part of the game they get 2 minutes to collect credits that they can use later on in the game. There are three items that they can collect to earn credits, worth 14, 25 and 36 points (use an appropriate combination).
Give the characters Matt and Jane a selection of items and ask, "Who has collected the most credits?"
Estimate first. What's your first thinking about this? Why? Take a class vote. Prove it!

Extension Question 1: The second time he played, Matt thought his credits added up to 400 (or some suitable number). Is that possible? Prove it!

Extension Question 2: Jane wonders if each of her credits has an even number value is it possible for her total score to be an odd number? What if one of the credits has an odd number value and two credits have an even number value. Now is it possible? How? Prove it!

This problem facilitates a composite class working on the same problem because it has multiple entry points. The values of the credits can easily be changed and children can be involved in selecting values that they feel are appropriate for them.

It is possible to be successful in finding a solution to this problem through using: additive or multiplicative thinking. Notice the similarity in this problem solving question from Foundation to year 10.

Look at the following calculations:
4 x 2 + 2
2 + 2 x 4
If you use the same values and operations, but in a different order, do you always get the same answer? Investigate.

How could you work out the value of 15 x 16 without using the algorithm?

Notice the use of this question in each proficiency.

It is important for students to first experience questions such as this in a problem solving context, where there is no prior knowledge of a process that would work. Hence, exploration of possible approaches and checking of the validity of the solution is demanded.

As with all problem solving questions, this question gives students opportunities to reason and hence build understanding.

Problem solving questions can be detailed, but they can also be very brief. Using the language of 'first thinking', implies that more thinking will be done and you may well change your mind. Keeping a record of changing thoughts is an important part of students being able to observe HOW they learn. Some students find it difficult to keep a record of ideas that they no longer believe to be true, preferring to erase their initial thoughts. If the teacher makes it clear that they are marking the students THINKING, not their final answer, then erasing their changing ideas is erasing the part that the teacher wants to see evidence of. Congratulating students for changing their mind in light of new information, rather than just congratulating them when they reach an answer, will help to build a disposition of sharing ideas.

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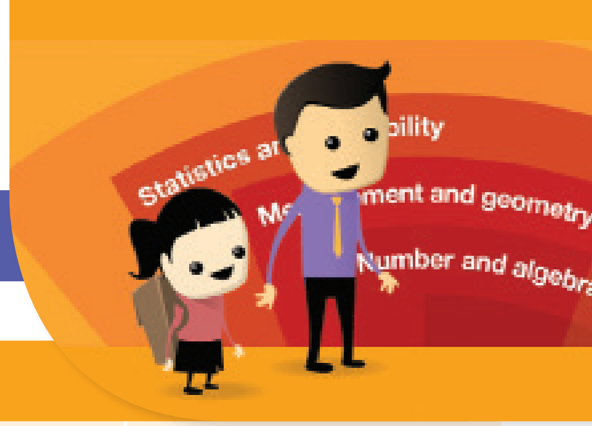

Understanding: Years 5-6

What patterns/connections/relationships can you see?

This is about noticing and using the characteristics of shapes, objects, quantities and patterns that show similarity and difference. It is about looking for patterns and connections in number, in shape, and in data.

As students move from year 3 to year 6 we support them to make generalisations (detailed in the reasoning proficiency) from the patterns that they notice.

Noticing similarity and difference helps students to build conceptual understanding.

Pedagogical questions:

- How are these... (values/shapes/angles/questions/graphs/words/number sentences) the same as each other?
- How are these... (values/shapes/angles questions/graphs/words/number sentences) different to each other?
- What's the connection between…?
- Which is the odd one out?
- What if... (change something), is it still…?
- Is it always the same? Are there any exceptions?
- Estimate…
- Which is greater/bigger/larger/taller?*
- Which is less/smaller/shorter?*

** Asking closed questions such as these can allow the teacher to see the connections that the student is/is not making, even if the student can't articulate the connections. These questions can help the teacher to identify the root of the misconception.*

Examples

How is 3 x 4 the same as 4 x 3 and how is 3 x 4 different to 4 x 3?

What's the connection between these calculations?
7 x 6 = 42
3½ x 12 = 42
(The answer is not just that they both have an answer of 42!)

How are these number sequences the same as each other? How are they different to each other? Describe a rule for generating each of these number sequences. How are your rules the same as each other? How are they different?

3½, 6½, 9½, 12½, 15½…
4.1, 7.1, 10.1, 13.1, 16.1…
1¾, 4¾ , 7¾, 10¾, 13¾…
2.25, 5.25, 8.25, 11.25, 14.25…
You could provide students with the structure; 'Start with… and add on…' if they need support.

12 has an even amount of factors. 15 has an even amount of factors. 12 is an even number, 15 is an odd number. Will all numbers have an even amount of factors?

Can you answer backwards/inverse questions?

This is about working flexibly with a concept.

Pedagogical questions:

- If the answer is… what might the question have been?
- What's missing in this number sentence/from this group/in this pattern?

Examples

23 x   = 1311
34 + 21 = 40 +

I'm thinking of a rectangle. Its area is 24cm. The length is 6cm. What is the width?

One third of a class brought lunch today. If 9 students brought lunch, how many students are in this class?
In year 5 it would be appropriate for students to investigate questions such as this in a problem solving situation in order to build understanding. Hence, notice that questions similar to this are in the problem solving proficiency.

Can you represent or calculate in different ways?

This is about representing amounts, patterns, shapes and data in different ways.

This is also about finding different ways to calculate the answer to computation problems involving all four operations (addition, subtraction, multiplication and division).

Pedagogical questions:

- What is another way…?
- What is another way to represent that?
- What is another way to work that out?
- What is another way to check that?
- What is another way to do that calculation?
- Simplify…
- Rename…
- Represent… in multiple ways.

Examples

Is there more than one way to work out 15 x 16?
Notice the use of this question in each proficiency.

Which of the following fractions can you simplify?
18/30   18/29   13/52   7/21

How would you represent 'even's chance' as a fraction, decimal and percentage?

Reasoning: Years 5-6

In what ways can you prove…?

This is really about convincing yourself and others about your mathematical thinking.

Across years 3 to 6; proof, like understanding, will be represented using models, diagrams and symbols/calculations. The complexity of the diagrams will develop to be more abstract in appearance and will use mathematical conventions, such as those for labelling angles or communicating length.

It is important to evaluate different ways of proving the same idea and justify the choices that are made.

Pedagogical questions:

- Prove that…
- Convince me, yourself, someone who thinks differently…
- Don't ask IF you are correct; tell when you KNOW you're correct, and tell HOW you know.
- What else could it be?
- Why is that the best way to show…?

Examples

Meg says that 14 x 17 will have the same answer as 15 x 16. Why do you think that Meg has made this connection? Do you agree/disagree? Prove it!

In what ways can you communicate?

This is about making thinking visible and sharing your ideas using mathematical terminology, diagrams and symbolic representations.

It is important to evaluate different ways to communicate the same idea.

Pedagogical questions:

- What's the best way to record your results and why?
- How come…?
- Explain it/why? (to somebody who hasn't been involved in the learning).
- Can you show me how that works?
- Why did you choose to…?
- Why is it not… (followed by an incorrect name or process)?
- Why can't I… (followed by an incorrect name or process)?

Examples

Communication of mathematical ideas can be emphasised in any proficiency, with any content.

In what ways can your thinking be generalised?

This is very strongly connected to looking for patterns and relationships. This is about making statements that describe a pattern that always exists. At this stage, generalisations will often be written. Across years 5 and 6, age appropriate mathematical terminology will be used.

Generalisations are certainties. They will always be true and it is not necessary to collect further information.

Pedagogical questions:

- Why are these always the same/different?
- Is there a rule that we could use to describe…?
- Is there a rule that always works?
- What makes these different processes the same?
- Use sentence structures, like the following, as writing frames to encourage generalisation:
  - The bigger/smaller the…
  - The older/younger the…

Examples

Draw a rectangle and enlarge it with a scale factor of two. What do you notice about the area of the enlarged rectangle compared to the area of the original rectangle? Start with a rectangle of a different area. What do you notice about the area of the enlarged rectangle compared to the area of the original rectangle? Exchange some results with a peer. Is there a rule about how the area of the enlarged rectangle compares to the area of the original? Does this rule work if you enlarge your rectangle by a scale factor of three?
Enlargement is a year 5 content description, but investigating it in this way brings in square numbers, which is a year 6 content description.

What can you infer?

This is about developing logical thought processes. These processes sometimes follow the structure: if…, then… This type of thinking helps to create new information from known information.

Logical thought can also be about working out a set of possibilities and narrowing them down as you get more information.

We make an inference when the known information suggests a particular connection or connections. Further information/testing may show an initial inference to be true or false.

Pedagogical questions:

- Now that you know… can you work out…?
- I'm thinking of... (a number/a shape) and I'm going to give you some clues… Can you work out what my number/shape is?
- I'm thinking of... (a number/a shape) and I'm going to give you some clues… Can you work out what the possible answers are?
- I'm thinking of... (a number/a shape). You can ask questions to help you to work out what it is, but I can only answer yes or no.
- You could use sentence structures such as:
  - If… then…
  - Because I know… I also know…

Examples

The arm spans of six people were recorded. Here are the measurements: 63, 70, 65, 68, 71, 64. What units do you think were used? What additional information would you like? What do you think if I tell you that they are people of school age?

I'm thinking of a fraction between 0 and 1. I can answer yes or no to your questions. You can't ask the same type of question more than once. Can you work out what my fraction is?
The first time you try a question like this, students will probably need support to try to think about different styles of questions that they could ask. Examples are: Is it greater than ½? Is it smaller than ¼? Is it a unit fraction? Is the denominator an odd number? Is it equivalent to 2/6? Would I need more than 6 of them to make a whole?