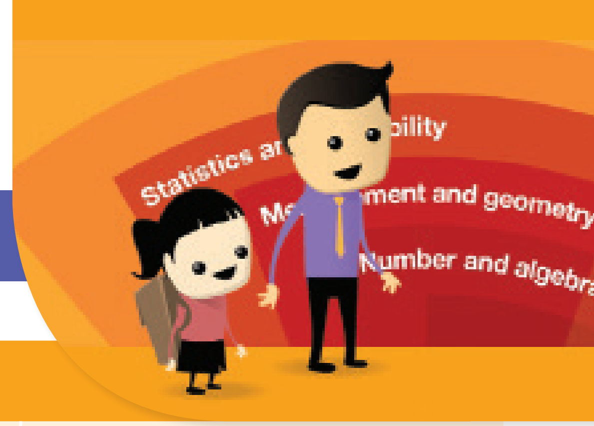
The BitL tool - mathematics years 3-4



Fluency: Years 3-4

What can you recall?

This is about remembering/identifying mathematical, names, shapes, symbols, facts and processes that are important to know when working with mathematical ideas.

Pedagogical questions:

- How could you record that mathematically?
- How could you... (eg calculate that)?
- How could you use a calculator to…?
- Can you remember a way to…?
- What is the value of… (a calculation that you would expect automatic recall of eg number pairs to 10, to 100, some times tables)?
- What is the name of…?
- What is the symbol for…?
- How many…?

Examples

What metric units of measurement are commonly used for length, area, volume, capacity and mass?

Recall multiplication and division facts up to and including 10 x 10.

Recall facts and definitions up to and including those used in the appropriate year level.

Can you remember a way to multiply a two digit number by a single digit number eg 36 x 7 ?

Can you remember a way to divide a two or three digit number by a single digit number (no remainders) eg 287 ÷ 7?

Notice that multiplication and division questions are also in the problem solving section. By the end of year 4 it would be appropriate for students to be able to respond to questions such as this as a fluency question. By this it is meant that students would be aware of several processes that they could apply. Prior to this it is crucial that students experience questions like this in a problem solving context, where they design possible processes and in doing so, make meaning of those processes.

Can you choose and use your mathematics flexibly?

To be able to choose and use mathematics efficiently students need to be able to recall processes and facts. Choosing and using is about selecting (age appropriate) processes, facts and mathematical language appropriate to the context.

Pedagogical questions:

- Choose a way to record that mathematically.
- Choose a way to... (count/estimate/rename/measure/compare/order/calculate/partition/rearrange/regroup/record/show/represent that).
- Use mathematical language to describe...
- What would be an efficient way to... (count/measure/order/compare/add on/subtract/multiply/divide/calculate/draw/record)?
- How could you... (partition/rearrange/regroup)?
- How could you use a calculator to check your answer?

Examples

Use an array to check your answer to the question 24 ÷ 3.

There are 24 children in Pam's class. Each child is allowed to bring up to 4 guests to their open day. What is the maximum number of guests that will be at the open day?

How could you use a calculator to work out the total number of books in three boxes with 36 books in each?

Problem solving: Years 3-4

Students benefit from working in a problem solving context in many aspects of the curriculum.

How can you interpret?

This is about creating meaning from the problem that has been presented or created by the student in response to curiosity about their world.

It is useful to have the students describe (in their own words) what they have been asked to do. Descriptions of the task could be oral or written, as appropriate for the students and the task.

Students should be encouraged to pose basic problems about their (immediate) world.

Pedagogical questions:

- What are you being asked to find out or show?
- What information is helpful?
- What information is not useful?

Closed questions can be useful to check if the student has accessed the information given in the question, for example
- How many…?
- How much…?
- When…?
(These questions will vary depending on the context of the problem)

In what ways can you model and plan?

This is about describing a problem mathematically. Across years 3 to 6 ideas are represented using models, pictures and symbols. The complexity of the pictures will develop from those representing an image of the problem (in years 3 and 4) to those that support thinking about the problem and are more abstract in appearance (in years 5 and 6).

It is important for students to think about how they will attempt to solve the problem, rather than rushing into taking measurements or making calculations without thinking first about how helpful that will be.

Pedagogical questions:

- Do you have an idea?
- What could you try?
- Have you done a problem like this one before?
- How could you test your idea?
- How might you start?
- Can you represent the problem as a picture or by using equipment?
- Would... (counting, a sum, a picture) help?
- Can you act it out?
- Can you represent the information using numbers and symbols?
- What questions could you ask?
- When we are being good problem solvers, what do we do to get started?
- Speak to someone who you think is being a good problem solver today. Ask them to show you what they are trying.

In what ways can you solve and check?

This is the mechanics of problem solving; the doing of calculations (the counting/adding/subtracting/sharing/grouping/building) and checking how appropriate the answer is.

Pedagogical questions:

- How can you... (add those numbers together/subtract that amount/multiply those amounts/divide those amount)?
- What processes could you try?
- Does that seem right to you?
- How can you check your answer?
- Do other people think that too?

Reflect

Students need to reflect on how reasonable their solution is - they should consider if they have made an appropriate interpretation in relation to the context of the problem.

There are different ways to solve problems and different ways to explain your thinking. At every stage of development, students benefit from sharing and reflecting on the strategies and reasoning of others.

Pedagogical questions:

- If the sharing is happening part-way through the problem solving process:
  - Would you like to change your mind and try something different?
- If the sharing is happening at the end of the problem solving process:
  - Would you use a different strategy next time?
  - How efficient was this strategy?
  - How reliable was this strategy?
  - Which was easiest for you to understand?
  - What did you like about...?
  - What would you do differently now?
  - How reasonable/realistic is your answer?

Examples

Teachers: Use your creative story telling skills to embellish these facts:

Matt and Jane have just finished playing a game. The winner is the person who has the highest score. The score is determined by the value of the counters they have collected. There are three colours of counters, worth 3, 4 and 6 (Use an appropriate combination up to 10 x 10). Give the characters Matt and Jane a selection of counters and ask, "Who won?".

Extension Question 1: Matt thinks his counters add up to 40 (or some suitable number) points. Is that possible? Prove it.

Extension Question 2: Jane wonders if each of her counters has an even number value. Is it possible for her total score to be an odd number? What if one of the counters has an odd number value and two counters have an even number value. Now is it possible? How? Prove it!

This problem facilitates a composite class working on the same problem because it has multiple entry points. The values of the counters can easily be changed and children can be involved in selecting values that they feel are appropriate for them. It is possible to be successful in finding a solution to this problem through using: skip counting strategies, additive or multiplicative thinking. Notice the similarity in this problem solving question from Foundation to year 10.

Which is the greater amount of time. 2 days 7 hours and 10 minutes OR fifty two and a half hours? Estimate first. What's your 'first thinking' about this? Why? Take a class vote about which is greater. Prove it!

Problem solving questions can be detailed, but they can also be very brief.

Using the language of 'first thinking', implies that more thinking will be done and you may well change your mind. Keeping a record of changing thoughts is an important part of students being able to observe HOW they learn. Some students find it difficult to keep a record of ideas that they no longer believe to be true, preferring to erase their initial thoughts. If the teacher makes it clear that they are marking the students THINKING, not their final answer, then erasing their changing ideas is erasing the part that the teacher wants to see evidence of. Congratulating students for changing their mind in light of new information, rather than just congratulating them when they get to an answer, will help to build a disposition of sharing ideas.

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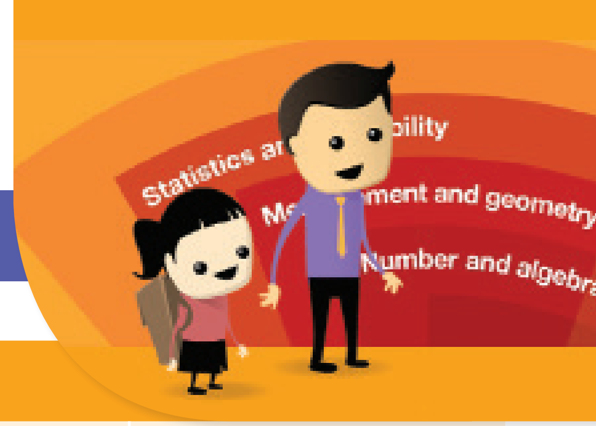

Understanding: Years 3-4

What patterns/connections/relationships can you see?

This is about noticing and using the characteristics of shapes, objects, quantities and patterns that show similarity and difference. It is about looking for patterns and connections in number, in shape, and in data.
As students move from year 3 to year 6 we support them to make generalisations (detailed in the reasoning proficiency) from the patterns that they notice.

Noticing similarity and difference helps students to build conceptual understanding.

Pedagogical questions:

- How are these... (values/shapes/angles/questions/graphs/words/number sentences) the same as each other?
- How are these... (values/shapes/angles/questions/graphs/words/number sentences) different to each other?
- What's the connection between…?
- Which is the odd one out?
- What if...(change something), is it still…?
- Which is greater/bigger/larger/taller?*
- Which is less/smaller/shorter?*

** Asking closed questions such as these can allow the teacher to see the connections that the student is/is not making, even if the student can't articulate the connections. These questions can help the teacher to identify the root of the misconception.*

Examples

How are 4 and 6 the same as each other? How are 5 and 3 the same as each other? How is the first pair (4 and 6) different to the second pair (5 and 3)?
NB: this is more than just naming numbers as odd and even. This is explaining the characteristics of odd and even.

How are these number sequences the same as each other? How are they different to each other? Describe a rule for generating each of these number sequences. How are your rules the same as each other? How are they different?
3 , 6 , 9 , 12 , 15…
4, 7, 10, 13, 16…
1½ , 4½, 7½, 10½, 13½…
2.5, 5.5, 8.5, 11.5, 14.5…
You could provide students with the structure; 'Start with…and add on…' if they need support. Obviously in this case the numbers that the sequences start with are all different, but the amount that is added on is always 3. Questions like these help students to see that add 3 sequences, for example, can look very different to each other.

What's the connection between these two number sequences?
2,4,6,8,10 and 1,3,5,7,9
There is more than one possible answer. They both go up in twos, the last number is 8 more than the first number, each number in the second sequence is one less than its partner in the first number sequence. Notice that this question is just another way of getting children to identify and describe similarities and differences.

What if I change the sum
34 + 59 to 33 + 60…
is it still the same?

Can you answer backwards/inverse questions?

This is about working flexibly with a concept.

Pedagogical questions:

- If the answer is… what might the question have been?
- What's missing in this number sentence/from this group/in this pattern?

Examples

I'm thinking of a multiplication sum and the answer to my sum is 12. What might the questions have been?

In ten minutes time it will be 3 o'clock. What time is it now?

Extension: In ten minutes my watch will show 3 o'clock, but I know that my watch is running 5 minutes too fast. What time is it now?
This extension question is still a 'flip or backwards' style question, but a slight change requires a higher level of reasoning. Notice that the content has remained the same, but the thinking has been extended.

If the answer is 12. What might the question have been? Is it possible to use each of the four operations in the question?

Can you represent or calculate in different ways?

This is about representing amounts, patterns, shapes and data in different ways, and calculating using different processes. This is also about finding different ways to calculate the answer to computation problems. In year 3, this would include addition, subtraction and multiplication. In year 4 this would also include division.

Pedagogical questions:

- What is another way…?
- What is another way to represent that?
- What is another way to work that out?
- What is another way to check that?
- What is another way to do that calculation?
- Rename…
- Represent…in multiple ways.

Examples

Rename 1250.
(1250 can be made from 1 thousand, 2 hundreds and 5 tens OR 12 hundreds and 5 tens etc).

Represent ½ in multiple ways.
Fractions of: a shape, a collection, an amount, a line.

Represent 36 in as many different arrays as possible.

Work out 287 ÷ 7 in two (or more) different ways.
By the end of year 4 we would expect students to have a method to be able to answer questions such as this, but students benefit from having different suitable approaches, so that they can begin to choose the most appropriate approach for a particular calculation. For this question a student may reason that 7 x 10 =70 and 70 x 4 =280, so there are 40 7's in 280 and so there would be 41 7's in 287.

Reasoning: Years 3-4

In what ways can you prove…?

This is about convincing yourself and others about your mathematical thinking.

At this stage proof would involve using equipment, diagrams and simple calculations.

It is important to evaluate different ways of proving the same idea and justify the choices that are made.

Pedagogical questions:

- Prove that…
- Convince me, yourself, someone who thinks differently…
- Try not to ask IF you are correct, but instead try to tell when you KNOW that you are correct. Then share HOW you know.
- What else could it be?

Examples

Connected to the problem solving question:
'134 can be made from 1 hundred, 3 tens and 4 ones. If you only had tens and ones, could you still build 134?'
The reasoning (proof) emphasis would be:
Prove that your different solutions are worth the same as each other.

In what ways can you communicate?

This is about making thinking visible and communicating a logical progression of ideas.

It is important to evaluate different ways to communicate the same idea.

Pedagogical questions:

- What's the best way to record your results, and why?
- How come…?
- Explain it/why? (to somebody who hasn't been involved in the learning, eg parent, a child in a different class).
- Can you show me how that works?
- Why did you choose to…?
- Why is it not… (followed by an incorrect name or process)?
- Why can't I… (followed by an incorrect name or process)?

Examples

Communication of mathematical ideas can be emphasised in any proficiency, with any content.

In what ways can your thinking be generalised?

This is really strongly connected to looking for patterns and relationships. This is about making statements that describe a pattern that always exists. At this stage generalisation may be verbal or written. Across years 3 and 4, age appropriate mathematical terminology will be used increasingly.

Pedagogical questions:

- Why are these always the same/different?
- Is there a rule that we could use to describe…?
- Is there a rule that always works?
- What makes these different processes the same?

Examples

What is it about all odd numbers that makes them the same and all even numbers the same?

So what makes the odd numbers different to the even numbers?

What can you infer?

This is about developing logical thought processes. These processes sometimes follow the structure: if…, then… This type of thinking helps to create new information from known information.

Logical thought can also be about working out a set of possibilities and narrowing them down as you get more information.

Pedagogical questions:

- Now that you know… can you work out…?
- I'm thinking of... (a number/a shape) and I'm going to give you some clues… Can you work out what my number/shape is?
- I'm thinking of... (a number/a shape) and I'm going to give you some clues… Can you work out what the possible answers are?
- I'm thinking of... (a number/a shape). You can ask questions to help you to work out what it is, but I can only answer yes or no.
- You could use sentence structures such as:
  - If… then…
  - Because I know… I also know…

Examples

I am thinking of a number. My number is between one thousand and two thousand. It uses the digits 0, 1, 3, and 6.

What could my number be?
If I say that my number is odd, what could it be now?

Can you ask a question that would help you to identify my number?

I'm thinking of 2 numbers. When I multiply these numbers together I get 12. When I add them together I get an even number.
x = 12
\+ = even number

What do you think my two numbers could be? Are you certain or do you want more information?
This question requires inference, but you could also emphasise the need for clear communication of thinking.